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AN APPROACH TO GROUP DECISION MAKING BASED ON 2-DIMENSION UNCERTAIN LINGUISTIC INFORMATION

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Abstract. Based on the definition of 2-dimension linguistic information of multiple attribute decision making problems proposed by Zhu, Zhou and Yang (2009), the information on evaluation is extended to 2-dimension uncertain linguistic variables, and a new method is proposed to solve the multiple-attribute group decision making problems in which the attribute values take the form of 2-dimension uncertain linguistic variables and the attribute weights are unknown. Firstly, the II class of uncertain linguistic information is transformed into the subjective weights of the experts, and then the subjective weights, the similarity degree of experts' evaluation information and authority weights are aggregated to the comprehensive weights of each expert. By the comprehensive weights, the group decision making matrix is produced by weighting evaluation information of each expert. Then the maximum deviation method is used to calculate the attribute weights and TOPSIS method is proposed to rank the alternatives. Finally, an example is given to illustrate the decision-making steps and the effectiveness of this method.

Keywords: 2-dimension uncertain linguistic variables, multiple attribute group decision making, TOPSIS.

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1. Introduction

Multiple attribute decision making (MADM) has been extensively applied to various areas ranging from economics to engineering technology. However, for some decision-making problems, such as personality assessment, automotive performance evaluation, etc. (Yan *et*

al. 2011; Han, Liu 2011; Kaya, Kahraman 2011), the decision makers often give the evaluation information as the linguistic terms directly, such as good, medium good, medium, medium poor and poor, etc. For example, when we want to make a decision-making about buying a car, usually, we may consider the price, performance, appearance, comfort, etc. Except that the price can be expressed by quantitative data, but for performance, appearance and comfort we can give the evaluation information by the linguistic terms. For instance, we can think that a car's performance is good, and another's is medium. Here, "good" and "medium" belong to the linguistic terms. If the decision making environment is more fuzzy and uncertain, the decision makers also use the interval linguistic value or the uncertain linguistic to express the evaluation information. Some achievements have been made in the study of the MADM problems based on the uncertain linguistic variables (Zhang *et al.* 2006, 2007; Cheng *et al.* 2006; Xu 2006c, 2006d; Xu *et al.* 2007; Wei *et al.* 2007; Liu 2011; Liu, Su 2010; Liu, Wang 2011; Liu, Zhang 2010, 2011a, 2011b; Liu *et al.* 2011). For example, Zhang *et al.* (2007) studied the MADM problems in which the attribute value is the uncertain linguistic variables and the attribute weight is known, and proposed the UEWAA operator to aggregate the group decision making information and utilized the probability matrix ranking formula to rank the order of the alternatives; Zhang *et al.* (2006) studied the multiple attribute group decision making problems in which the attribute values and attribute weights also take the form of linguistic information, and proposed the ULWM operator to aggregate the comprehensive attribute values of the alternatives and ranked the alternatives based on ranking the fuzzy complementary judgment matrix. Cheng *et al.* (2006) studied the multiple attribute decision making problems in which the information on evaluation is the uncertain linguistic information and the decision making matrix has some unknown values, and proposed a method which can fill the unknown values in linguistic decision making matrix and a method which can determine the comprehensive attribute weights based on attribute subjective and objective weights. Xu *et al.* (2007) studied the multiple attribute decision making problems in which the attribute value is the uncertain linguistic variables and the attribute weight is unknown, and proposed the projection model to determine the attribute weigh, and then utilized the probability matrix ranking formula to rank the alternatives. Xu (2008b) investigated group decision making problems with multiple types of linguistic preference relations, and proposed a method to reach consensus among the individual preferences and the group's opinion. Wei *et al.* (2007) studied the multiple attribute group decision making problems with uncertain linguistic information, in which the attribute weight and expert weight take the form of real numbers, and the preference value takes the form of uncertain linguistic variables, and proposed a new method based on the ULWGM and the ULHGA operators. Xu (2009) defined some unbalanced linguistic label sets, and then developed some transformational functions to unify the given multigranular linguistic labels in a unique linguistic label set without lossing the information. Moreover, he utilized the uncertain linguistic weighted averaging operator to aggregate all individual uncertain linguistic decision matrices into a collective one, and defined two measures for similarity: one for measuring the similarity degree between each pair of uncertain linguistic variables, and the other for checking the degrees of consensus among the individual uncertain linguistic decision matrices and the collective uncertain linguistic decision matrix. Finally, an interactive approach to MAGDM with multigranular uncertain linguistic information was proposed. Xu (2010) proposed that

the uncertain linguistic weighted geometric mean operator is utilized to aggregate all the individual uncertain multiplicative linguistic preference relations into a collective one, and then a simple approach is developed to determine the experts' weights by utilizing the consensus degrees among the individual uncertain multiplicative linguistic preference relations and the collective uncertain multiplicative linguistic preference relations. Furthermore, a proposal for a practical interactive procedure for group decision making is given based on uncertain multiplicative linguistic preference relations.

There are another decision making problems in the real decision making situation, for example, the evaluation on the projects or awards for the science and technology, the peer review of the master and doctor thesis or papers, etc. The decision makers not only make the conclusions on evaluation, but also show the evaluation reliability of themselves with the familiarity and other forms. For such decision making problems, Zhu *et al.* (2009) proposed the definition of the 2-dimension linguistic information to use I and II class of the linguistic evaluation information to describe the evaluation of decision makers with respect to the evaluation objects, where the I class of the linguistic evaluation information is used to describe the decision making objects, and the II class is used to describe the subjective evaluation of the reliability of the decision results. Finally, a group decision making method based on the evidence combination rules is given. However, the method proposed by Zhu *et al.* (2009) was meant to deal with single attribute decision making problems, but not to solve the multiple attribute problems, and the decision making method was more complex.

Based on the definition of 2-dimension linguistic information proposed by Zhu *et al.* (2009), the evaluation information is extended to 2-dimension uncertain linguistic variables and multiple attribute decision making problems, and a new method is proposed to solve the multiple attribute group decision making problems in which the attribute values take the form of 2-dimension uncertain linguistic variables and the attribute weight is unknown. Firstly, the II class of uncertain linguistic information is transformed into the subjective weight of the experts, and the similarity degree of experts' evaluation information and authority weights of each expert are aggregated to the comprehensive weights of each expert in different attributes and alternatives, the comprehensive weights can be used to aggregate each expert's evaluation information into the group decision making matrix. Then, the maximum deviation method is used to calculate the attribute weights, and TOPSIS method is proposed to rank the alternatives. Finally, an example is given to illustrate the decision-making steps.

2. The description and the operation rules of the uncertain linguistic information

Support that $S = (s_0, s_1, \cdots, s_{l-1})$ is a pre-defined and ordered linguistic term set with odd elements, and S should satisfy the following properties (Xu 2004a, 2004b; Xu 2006a):

1. The set is ordered: $s_i \prec s_j$, if and only if $i < j$,
2. There is the negation operator: $neg(s_i) = s_{l-i}$,
3. Maximum operator: $\max(s_i, s_j) = s_i$, if $i \geq j$,
4. Minimum operator: $\min(s_i, s_j) = s_i$, if $i \leq j$.

In practice, let l be equal to 3, 5, 7, 9, etc. it can be defined as:

$S=(s_0,s_1,s_2)$ = (poor, fair, good).

$S=(s_0,s_1,s_2,s_3,s_4)$ = (very poor, poor, fair, good, very good).

$S=(s_0,s_1,s_2,s_3,s_4,s_5,s_6)$ = (very poor, poor, slightly poor, fair, slightly good, good, very good).

$S=(s_0,s_1,s_2,s_3,s_4,s_5,s_6,s_7,s_8)$ = (extremely poor, very poor, poor, slightly poor, fair, slightly good, good, very good, extremely good).

In the process of information aggregation, however, some results may not exactly match any linguistic labels in S. To preserve all the given information, the discrete term set S is extended to a continuous term set $\overline{S}=\{s_\alpha \mid \alpha\in[0,q]\}$, where s_α meets all the characteristics above and $q(q\succ l)$ is a sufficiently large positive integer. If $s_\alpha\in S$, then we call s_α the original term, otherwise, we call s_α the virtual term. In general, the decision maker uses the original linguistic terms to evaluate alternatives, and the virtual linguistic terms can only appear in calculation and ranking (Xu 2004b, 2004c).

Definition 1 (Xu 2006a): Let $\tilde{s}=[s_a,s_b]$, where $s_a,s_b\in\overline{S}$ and $a\le b$, and s_a and s_b are the lower and the upper limits respectively, then we call $\tilde{s}$ an uncertain linguistic variable.

Suppose that the set $\tilde{S}$ is composed of all the uncertain linguistic variables, and $\tilde{s}_1=[s_{a1},s_{b1}]$ and $\tilde{s}_2=[s_{a2},s_{b2}]$ are any two uncertain linguistic variables, and $\lambda_1\in[0,1]$ and $\lambda_1\in[0,1]$, then their operation rules are shown as follows (Xu 2004b, 2006a):

1. $\tilde{s}_1\oplus\tilde{s}_2=[s_{a1},s_{b1}]\oplus[s_{a2},s_{b2}]=[s_{a1+a2},s_{b1+b2}]$, (1)
2. $\tilde{s}_1\otimes\tilde{s}_2=[s_{a1},s_{b1}]\otimes[s_{a2},s_{b2}]=[s_{a1\times a2},s_{b1\times b2}]$, (2)
3. $\tilde{s}_1/\tilde{s}_2=[s_{a1},s_{b1}]/[s_{a2},s_{b2}]=[s_{a1/b2},s_{b1/a2}]$, *if* $a2\ne 0,b2\ne 0$, (3)
4. $\lambda\tilde{s}_1=\lambda[s_{a1},s_{b1}]=[s_{\lambda^* a1},s_{\lambda^* b1}]$, (4)
5. $\lambda(\tilde{s}_1\oplus\tilde{s}_2)=\lambda\tilde{s}_1\oplus\lambda\tilde{s}_2$, (5)
6. $(\lambda_1+\lambda_2)\tilde{s}_1=\lambda_1\tilde{s}_1\oplus\lambda_2\tilde{s}_1$. (6)

Definition 2 (Xu 2006b; Liu, Zhang 2009): Let $\tilde{s}_1=[s_{a1},s_{b1}]$ and $\tilde{s}_2=[s_{a2},s_{b2}]$ be any two uncertain linguistic variables, then the distance of the $\tilde{s}_1$ and $\tilde{s}_2$ is defined as follows:

$$d(\tilde{s}_1,\tilde{s}_2)=\sqrt{\left(a2-a1\right)^2+\left(b2-b1\right)^2\Big/2}\,. \tag{7}$$

Definition 3 (Xu 2005, 2009): If $\tilde{s}_1=[s_{a1},s_{b1}]$ and $\tilde{s}_2=[s_{a2},s_{b2}]$ are any two uncertain linguistic variables, and l is the number of the linguistic variables in the linguistic variables set S, then the similarity degree of the $\tilde{s}_1$ and $\tilde{s}_2$ is defined as follows:

$$s(\tilde{s}_1,\tilde{s}_2)=1-\frac{abs(a1-a2)+abs(b1-b2)}{2\times\left(l-1\right)}\,. \tag{8}$$

3. The decision making method

3.1. The description of the decision making problems

If $A=(a_1,a_2,\cdots,a_m)$ is the set of alternatives, and $C=(c_1,c_2,\cdots,c_n)$ is the set of attributes. $W=(w_1,w_2,\ldots,w_n)$ is the weight vector of attributes $c_j(j=1,2,\cdots,n)$ and it is unknown, but

meets $0 \le w_j \le 1$, $\sum_{j=1}^{n} w_j = 1$. If $\left(e_1, e_2, \cdots, e_p\right)$ is the experts set in the group decision making, and $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_p)$ is the authority weight set of the experts, where $0 \le \lambda_k \le 1, \sum_{k=1}^{p} \lambda_k = 1$. Suppose that $\left([x_{ij}^{Lk}, x_{ij}^{Uk}], [g_{ij}^{Lk}, g_{ij}^{Uk}]\right)$ is the attribute value of the attribute c_j in the alternative a_i, given by the expert e_k, which takes the form of the 2-dimension uncertain linguistic information, where $[x_{ij}^{Lk}, x_{ij}^{Uk}]$ is the I class of the uncertain linguistic evaluation information, and it shows the object evaluation given by the expert, and x_{ij}^{Lk} and x_{ij}^{Uk} are the elements of the pre-defining linguistic evaluation set $S_1 = (s_0, s_1, \cdots, s_{l-1})$, and $[g_{ij}^{Lk}, g_{ij}^{Uk}]$ is the II class of the uncertain linguistic evaluation information, and it shows the subjective evaluation of the reliability of the decision results, and g_{ij}^{Lk} and g_{ij}^{Uk} are the element of the pre-defining linguistic evaluation set $S_2 = (s_0, s_1, \cdots, s_{t-1})$. Based on these conditions, the order of the alternatives of multiple attribute decision making problems based on the 2-dimension uncertain linguistic variables can be ranked.

3.2. The decision making method

3.2.1. Transform the II class of the uncertain linguistic evaluation information to the subjective weight of the decision makers

The II class of the uncertain linguistic evaluation information shows the subjective evaluation of the reliability of the decision results. No one knows the reliability of the subjective evaluation better than the decision makers themselves, under the premise that the rational, knowledge and experience level of the decision makers can satisfy the requirement of the decision making. Therefore, we can transform the II class of the uncertain linguistic evaluation information, given by the decision makers, to the subjective weight of the decision makers. The larger the evaluation value of the II class of the linguistic evaluation information is (the more faith the decision maker has), and the larger the weight of decision maker is, and vice versa.

1. The UL-OWA operator

Definition 4 (Yager 2004): Let function $\rho : [0,1] \to [0,1]$ satisfy:

1. $\rho(0) = 0$;
2. $\rho(1) = 1$;
3. if $x > y$, then $\rho(x) > \rho(y)$.

Then ρ is called the basic unit-interval monotonic function (the BUM function).

Definition 5 (Zhang, Xu 2005): If $[s_a, s_b]$ is the uncertain linguistic variable, and

$$f_\rho\left(\left[s_a, s_b\right]\right) = s_{\int_0^1 \frac{d\rho(y)}{dy}\left(b - y\left(b-a\right)\right)dy}, \tag{9}$$

then f is called the uncertain linguistic variable OWA operator (the UL-OWA operator).

If $\rho(y) = y^\delta (\delta \ge 0)$, then $f_\rho([s_a, s_b]) = s_{\frac{b+\delta a}{\delta+1}}$.

2. Transform the II class of the uncertain linguistic evaluation information to the linguistic information

We utilize the UL-OWA operator to transform the II class of the uncertain linguistic variable $[g_{ij}^{Lk}, g_{ij}^{Uk}]$ to the linguistic variable $g_{ij}^{k} = f_{\rho}[g_{ij}^{Lk}, g_{ij}^{Uk}]$.

3. Calculate the weight ξ_{ij}^{k} of the attribute c_j under the alternative a_i given by the expert e_k

$$\xi_{ij}^{k} = \frac{g_{ij}^{k}}{\sum_{j=1}^{n} g_{ij}^{k}}. \tag{10}$$

3.2.2. Calculate the relative similarity degree of decision making information of the experts

First, we should calculate the relative similarity degree of the evaluation value of the attribute c_j in the alternative a_i, given by any two experts e_k and e_q. The formula (8) can be used to calculate the relative similarity degree $S_{ij}(k,q)$ of any two uncertain linguistic variables $[x_{ij}^{Lk}, x_{ij}^{Uk}]$ and $[x_{ij}^{Lq}, x_{ij}^{Uq}]$ given by the experts e_k and e_q. Then we should calculate the average similarity degree in the expert group of the evaluation value of the attribute c_j in the alternative a_i, given by the expert e_k:

$$AS_{ij}(k) = \frac{1}{p-1} \sum_{q=1, q\neq k}^{p} S_{ij}(k,q). \tag{11}$$

Based on these, we can calculate the relative similarity degree in the expert group of the evaluation value of the attribute c_j in the alternative a_i, given by the expert e_k:

$$RS_{ij}(k) = \frac{AS_{ij}(k)}{\sum_{k=1}^{p} AS_{ij}(k)}. \tag{12}$$

3.2.3. Determine the comprehensive weight of experts

We should comprehensively consider these conditions, such as the similarity degree of the expert evaluation information and the expert group evaluation information, the weight transformed by the II class of the uncertain linguistic evaluation information and the authority weight of the experts.

The multiplication synthesis method has the multiplier effect (Zeng 1997). So we aggregate the similarity degree of the expert evaluation information and the expert group evaluation information, and the weight transformed by the II class of the uncertain linguistic evaluation information, then we can get the comprehensive weight.

$$\gamma_{ij}^{k} = \frac{\xi_{ij}^{k} \times RS_{ij}(k)}{\sum_{k=1}^{p} \xi_{ij}^{k} \times RS_{ij}(k)}. \tag{13}$$

We further aggregate the authority weight of the experts, and we finally get the comprehensive weight of the attribute c_j in the alternative a_i, given by the expert e_k.

$$\omega_{ij}^k = \alpha \times \lambda_k + (1-\alpha) \times \gamma_{ij}^k, \tag{14}$$

where α is the weight coefficient, and $0 \le \alpha \le 1$. The value of α shows the preference of the expert.

3.2.4. Aggregate the expert's evaluation information

We should aggregate the expert's evaluation information, and transform the 2-dimension linguistic evaluation information into 1-dimension linguistic evaluation information $Z = \left[z_{ij} \right]_{m \times n}$, where $z_{ij} = \left[z_{ij}^L, z_{ij}^U \right]$, then:

$$z_{ij}^L = \sum_{k=1}^{p} \left(\omega_{ij}^k \times x_{ij}^{Lk} \right), z_{ij}^U = \sum_{k=1}^{p} \left(\omega_{ij}^k \times x_{ij}^{Uk} \right). \tag{15}$$

3.2.5. Calculate the attribute weight based on the maximizing deviations method

The attribute weight is unknown, and the uncertainty of the attribute weight will result in the uncertainty of the ranking order of the alternatives. In general, if the attribute value $z_{ij}(j = 1, 2, \cdots, n)$ among all the alternatives are little different with respect to attribute c_j, it shows that the attribute c_j plays a less important role in the decision making procedure, and the smaller weight will be given. Contrariwise, if the attribute c_j makes the attribute values $z_{ij}(j = 1, 2, \cdots, n)$ among all the alternatives have obvious differences, such an attribute plays an important role in choosing the best alternative. Therefore, while ranking the alternatives, the larger the deviation of the attribute value of the alternative is, the larger its weight is, and vice versa (Zhou, Liu 2007).

For the attribute c_j, the deviation value of alternative A_i to all the other alternatives can be defined as $D_{ij}(w_j) = \sum_{l=1}^{m} d(z_{ij}, z_{lj}) w_j$, then $D_j(w_j) = \sum_{i=1}^{m} D_{ij}(w_j) = \sum_{i=1}^{m} \sum_{l=1}^{m} d(z_{ij}, z_{lj}) w_j$ and represents the total deviation value of all alternatives to the other alternatives for the attribute c_j. $D(w_j) = \sum_{j=1}^{n} D_j(w_j) = \sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{l=1}^{m} d(z_{ij}, z_{lj}) w_j$ represents the deviation of all attributes to all alternatives. The optimization model is constructed as follows (Xu 2008a):

$$\begin{cases} \max D(w_j) = \sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{l=1}^{m} d(z_{ij}, z_{lj}) w_j \\ s.t \sum_{j=1}^{n} w_j^2 = 1, w_j \ge 0, j = 1, 2 \ldots\ldots n \end{cases}. \tag{16}$$

We can get the normalized attribute weight based on this model:

$$w_j = \frac{\sum_{i=1}^{m}\sum_{l=1}^{m} d(z_{ij}, z_{lj})}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{l=1}^{m} d(z_{ij}, z_{lj})}. \tag{17}$$

3.2.6. Determine the order of the alternatives based on TOPSIS

TOPSIS (Technique for Order Performance by Similarity to Ideal Solution) method is the famous multiple attribute decision making method, proposed by the Hwang and Yoon (1981). This method is usually used to solve such multiple attribute decision making problems when the attribute value is the real number. This paper utilizes the TOPSIS method to solve the multiple attribute decision making problems in which the attribute value is the uncertain linguistic variables, based on the distance formula between two uncertain linguistic variables.

1. Calculate the weight matrix:

$$V = [v_{ij}]_{m\times n} = \begin{bmatrix} \left[v_{11}^L, v_{11}^U\right] & \left[v_{12}^L, v_{12}^U\right] & \cdots & \left[v_{1n}^L, v_{1n}^U\right] \\ \left[v_{21}^L, v_{21}^U\right] & \left[v_{22}^L, v_{22}^U\right] & \cdots & \left[v_{2n}^L, v_{2n}^U\right] \\ \vdots & \cdots & \cdots & \vdots \\ \left[v_{m1}^L, v_{m1}^U\right] & \left[v_{m2}^L, v_{m2}^U\right] & \cdots & \left[v_{mn}^L, v_{mn}^U\right] \end{bmatrix},$$

where

$$v_{ij}^L = w_j z_{ij}^L, v_{ij}^U = w_j z_{ij}^U. \tag{18}$$

2. Calculate the positive and negative ideal solution of the weighted matrix:

$$V^+ = \left(v_1^+, v_2^+, \cdots, v_n^+\right) = \left(\left[v_1^{L+}, v_1^{U+}\right], \left[v_2^{L+}, v_2^{U+}\right], \cdots, \left[v_n^{L+}, v_n^{U+}\right]\right),$$

$$V^- = \left(v_1^-, v_2^-, \cdots, v_n^-\right) = \left(\left[v_1^{L-}, v_1^{U-}\right], \left[v_2^{L-}, v_2^{U-}\right], \cdots, \left[v_n^{L-}, v_n^{U-}\right]\right),$$

where

$$v_j^{L+} = \max_i(v_{ij}^L), v_j^{U+} = \max_i(v_{ij}^U), v_j^{L-} = \min_i(v_{ij}^L), v_j^{U-} = \min_i(v_{ij}^U). \tag{19}$$

3. Calculate the distance between each alternative and the positive and negative ideal solution:

$$D^+ = \left(d_1^+, d_2^+, \cdots, d_m^+\right) \text{ and } D^- = \left(d_1^-, d_2^-, \cdots, d_m^-\right),$$

where

$$\begin{cases} d_i^+ = \left[\sum_{j=1}^{n}\left(d(v_{ij}, v_j^+)\right)^2\right]^{1/2} \\ d_i^- = \left[\sum_{j=1}^{n}\left(d(v_{ij}, v_j^-)\right)^2\right]^{1/2} \end{cases}, \tag{20}$$

where $d(v_{ij}, v_j^+)$ is the distance between the uncertain linguistic variables v_{ij} and v_j^+, $d(v_{ij}, v_j^-)$ is the distance between the uncertain linguistic variables v_{ij} and v_j^-. They can be calculated by the formula (7).

4. Calculate the relative closeness degree of each alternative.

Let $C = (c_1, c_2, \cdots, c_m)$ be the relative closeness degree of each alternative, where

$$c_i = \frac{d_i^-}{d_i^+ + d_i^-} \quad (i = 1, 2, \cdots, m). \tag{21}$$

5. Rank the order of the alternatives

We can rank the order of the alternatives based on the values of the relative closeness degree. The lager the relative closeness degree is, the better the alternative is, vice versa.

4. An illustrate example

A practical use of the proposed approach involves the technological innovation ability to evaluate the four enterprises $\{a_1, a_2, a_3, a_4\}$, the attributes are shown as follows: the ability of innovative resources input (C_1), the ability of innovation management (C_2), the ability of innovation tendency (C_3) and the ability of research and development (C_4). Based on the four attributes, the three experts $\{e_1, e_2, e_3\}$ evaluated the technological innovation ability of the four enterprises. Supposedly $\lambda = (0.4, 0.32, 0.28)$ is the weight vector of the three experts, and the attribute values given by the experts take the form of 2-dimension uncertain linguistic variables, shown in Tables 1, 2 and 3. The experts utilize the I class of the linguistic evaluation set $S_1 = (s_0, s_1, s_2, s_3, s_4, s_5, s_6)$ and the II class of the linguistic evaluation set $S_2 = (s_0, s_1, s_2, s_3, s_4)$, and the attribute weight is unknown.

Table 1. The attributes' values with respect to four enterprises given by expert e_1

Enterprises	Attribute (C_1)	Attribute (C_2)	Attribute (C_3)	Attribute (C_4)
a_1	$([s_5, s_5], [s_2, s_3])$	$([s_2, s_3], [s_3, s_3])$	$([s_4, s_5], [s_4, s_4])$	$([s_3, s_4], [s_1, s_2])$
a_2	$([s_3, s_4], [s_2, s_3])$	$([s_5, s_5], [s_3, s_3])$	$([s_3, s_3], [s_4, s_4])$	$([s_4, s_4], [s_1, s_2])$
a_3	$([s_2, s_3], [s_2, s_3])$	$([s_3, s_4], [s_3, s_3])$	$([s_3, s_4], [s_4, s_4])$	$([s_4, s_5], [s_1, s_2])$
a_4	$([s_5, s_6], [s_2, s_3])$	$([s_1, s_2], [s_3, s_3])$	$([s_2, s_3], [s_4, s_4])$	$([s_3, s_4], [s_1, s_2])$

Table 2. The attributes' values with respect to four enterprises given by expert e_2

Enterprises	Attribute (C_1)	Attribute (C_2)	Attribute (C_3)	Attribute (C_4)
a_1	$([s_4, s_4], [s_3, s_4])$	$([s_3, s_4], [s_2, s_3])$	$([s_3, s_4], [s_3, s_3])$	$([s_5, s_6], [s_3, s_4])$
a_2	$([s_4, s_5], [s_3, s_4])$	$([s_2, s_3], [s_2, s_3])$	$([s_4, s_5], [s_3, s_3])$	$([s_2, s_3], [s_3, s_4])$

End of Table 2

Enterprises	Attribute (C_1)	Attribute (C_2)	Attribute (C_3)	Attribute (C_4)
a_3	$([s_3,s_4],[s_3,s_4])$	$([s_4,s_4],[s_2,s_3])$	$([s_2,s_3],[s_3,s_3])$	$([s_3,s_4],[s_3,s_4])$
a_4	$([s_5,s_5],[s_3,s_4])$	$([s_4,s_5],[s_2,s_3])$	$([s_1,s_2],[s_3,s_3])$	$([s_4,s_4],[s_3,s_4])$

Table 3. The attributes' values with respect to four enterprises given by expert e_3

Enterprises	Attribute (C_1)	Attribute (C_2)	Attribute (C_3)	Attribute (C_4)
a_1	$([s_5,s_5],[s_2,s_3])$	$([s_3,s_3],[s_2,s_2])$	$([s_4,s_4],[s_3,s_4])$	$([s_4s_5],[s_1,s_1])$
a_2	$([s_4,s_4],[s_2,s_3])$	$([s_4,s_5],[s_2,s_2])$	$([s_1,s_2],[s_3,s_4])$	$([s_3,s_3],[s_1,s_1])$
a_3	$([s_3,s_4],[s_2,s_3])$	$([s_5,s_5],[s_2,s_2])$	$([s_1,s_1],[s_3,s_4])$	$([s_4,s_4],[s_1,s_1])$
a_4	$([s_2,s_3],[s_2,s_3])$	$([s_2,s_3],[s_2,s_2])$	$([s_4,s_5],[s_3,s_4])$	$([s_4,s_5],[s_1,s_1])$

The evaluation steps used in this paper are proposed as follows:

1. **Transform the II class of the uncertain linguistic evaluation information to the subjective weight of the decision makers**

We select the BUM function $\rho(y) = y^2$, then the subjective weight is calculated as follows:

$$\xi^1 = \begin{bmatrix} 0.219\ 0.281\ 0.375\ 0.125 \\ 0.219\ 0.281\ 0.375\ 0.125 \\ 0.219\ 0.281\ 0.375\ 0.125 \\ 0.219\ 0.281\ 0.375\ 0.125 \end{bmatrix}, \quad \xi^2 = \begin{bmatrix} 0.278\ 0.194\ 0.250\ 0.278 \\ 0.278\ 0.194\ 0.250\ 0.278 \\ 0.278\ 0.194\ 0.250\ 0.278 \\ 0.278\ 0.194\ 0.250\ 0.278 \end{bmatrix}, \quad \xi^3 = \begin{bmatrix} 0.269\ 0.231\ 0.385\ 0.115 \\ 0.269\ 0.231\ 0.385\ 0.115 \\ 0.269\ 0.231\ 0.385\ 0.115 \\ 0.269\ 0.231\ 0.385\ 0.115 \end{bmatrix}.$$

2. **Calculate the relative similarity degree of the attribute c_j under the alternative a_i of each expert**

$$RS(1) = \begin{bmatrix} 0.344\ 0.328\ 0.328\ 0.321 \\ 0.328\ 0.346\ 0.375\ 0.317 \\ 0.313\ 0.333\ 0.327\ 0.328 \\ 0.354\ 0.333\ 0.375\ 0.328 \end{bmatrix}, \quad RS(2) = \begin{bmatrix} 0.313\ 0.328\ 0.328\ 0.321 \\ 0.328\ 0.288\ 0.313\ 0.333 \\ 0.344\ 0.350\ 0.365\ 0.328 \\ 0.375\ 0.292\ 0.333\ 0.344 \end{bmatrix}, \quad RS(3) = \begin{bmatrix} 0.344\ 0.344\ 0.344\ 0.357 \\ 0.344\ 0.365\ 0.313\ 0.350 \\ 0.344\ 0.317\ 0.308\ 0.344 \\ 0.271\ 0.375\ 0.292\ 0.328 \end{bmatrix}.$$

3. **Calculate the comprehensive weight**

Let the weight coefficient $\alpha = 0.4$, then the comprehensive weights are:

$$\omega^1 = \begin{bmatrix} 0.337\ 0.395\ 0.379\ 0.301 \\ 0.329\ 0.406\ 0.409\ 0.298 \\ 0.320\ 0.399\ 0.381\ 0.303 \\ 0.343\ 0.397\ 0.411\ 0.301 \end{bmatrix}, \quad \omega^2 = \begin{bmatrix} 0.333\ 0.291\ 0.274\ 0.442 \\ 0.342\ 0.270\ 0.266\ 0.450 \\ 0.351\ 0.302\ 0.293\ 0.446 \\ 0.374\ 0.272\ 0.277\ 0.457 \end{bmatrix}, \quad \omega^3 = \begin{bmatrix} 0.330\ 0.314\ 0.347\ 0.257 \\ 0.329\ 0.325\ 0.325\ 0.252 \\ 0.329\ 0.299\ 0.326\ 0.251 \\ 0.284\ 0.331\ 0.312\ 0.242 \end{bmatrix}.$$

4. Transform the 2-dimension information into 1-dimension linguistic evaluation information and aggregate into the group decision matrix

$$Z=\begin{bmatrix} [s_{4.667}, s_{4.667}] & [s_{2.605}, s_{3.291}] & [s_{3.726}, s_{4.379}] & [s_{4.141}, s_{5.141}] \\ [s_{3.671}, s_{4.342}] & [s_{3.867}, s_{4.461}] & [s_{2.617}, s_{3.208}] & [s_{2.848}, s_{3.298}] \\ [s_{2.680}, s_{3.680}] & [s_{3.899}, s_{4.299}] & [s_{2.056}, s_{2.730}] & [s_{3.554}, s_{4.303}] \\ [s_{4.148}, s_{4.775}] & [s_{2.146}, s_{3.146}] & [s_{2.348}, s_{3.348}] & [s_{3.699}, s_{4.242}] \end{bmatrix}.$$

5. Calculate the attribute weight based on the maximizing deviations method

$$W=(\ 0.248\ \ 0.275\ \ 0.246\ \ 0.231).$$

6. Calculate the weighted decision making matrix

$$V=\begin{bmatrix} [s_{1.158}, s_{1.158}] & [s_{0.716}, s_{0.905}] & [s_{0.917}, s_{1.078}] & [s_{0.954}, s_{1.185}] \\ [s_{0.911}, s_{1.078}] & [s_{1.064}, s_{1.227}] & [s_{0.644}, s_{0.790}] & [s_{0.656}, s_{0.760}] \\ [s_{0.665}, s_{0.913}] & [s_{1.073}, s_{1.182}] & [s_{0.506}, s_{0.672}] & [s_{0.819}, s_{0.992}] \\ [s_{1.030}, s_{1.185}] & [s_{0.590}, s_{0.865}] & [s_{0.578}, s_{0.824}] & [s_{0.853}, s_{0.978}] \end{bmatrix}.$$

7. Calculate the positive and negative ideal solution

$$V^{+}=\left([s_{1.158}, s_{1.185}]\ [s_{1.073}, s_{1.227}]\ [s_{0.917}\ , s_{1.078}]\ [s_{0.954}\ , s_{1.185}]\right),$$

$$V^{-}=\left([s_{0.665}, s_{0.913}]\ [s_{0.590}, s_{0.865}]\ [s_{0.506}\ , s_{0.672}]\ [s_{0.656}\ , s_{0.760}]\right).$$

8. Calculate the weight distance between each alternative and the positive and negative ideal solution

$$D^{+}=(0.340\ \ 0.500\ \ 0.595\ \ 0.554),$$

$$D^{-}=(0.680\ \ 0.488\ \ 0.455\ \ 0.401).$$

9. Calculate the relative closeness degree

$$C=(0.667\ \ 0.494\ \ 0.433\ \ 0.420).$$

10. Rank the order of the alternatives

Based on the value of the relative closeness degree, the order of each alternative is $a_1 \succ a_2 \succ a_3 \succ a_4$.

In order to verify the effectiveness of the method, we use the method proposed by Zhu *et al.* (2009). However, the method is only used with the linguistic variables and single attribute decision making problems. In order to apply it, we had to covert the uncertain linguistic variables to linguistic variables by averaging algorithm, then calculate utility value for each attribute with respect to each alternative, and weight the utility value for all attributes and get an integrated utility value for each alternative. Finally, rank the alternatives by the integrated utility values.

The calculated ranking is $a_1 \succ a_2 \succ a_3 \succ a_4$. There are the same results for two methods.

5. Conclusions

Multiple attribute group decision making based on the 2-dimension uncertain linguistic variables are widely used in the real decision making. Firstly, the II class of uncertain linguistic information is transformed into the subjective weights of the experts, and the similarity degree of experts' evaluation information and authority weights of each expert are aggregated to the comprehensive weights of each expert in different attributes and alternatives, the comprehensive weights can be used to aggregate each expert' evaluation information into the group decision making matrix. Then, the maximum deviation method is used to calculate the attribute weights, and TOPSIS method is proposed to rank the alternatives. Finally, an illustrate example is given to show the decision-making steps and the effectiveness of this method. This method proposed in this paper is easy to use and understand, and it enriched and developed the theory and method of 2-dimension uncertain linguistic multiple attribute decision making, and it provided the new idea to solve the 2-dimension uncertain linguistic multiple attribute decision making. In the future, we shall continue working in the extension and application of the developed method to other domains.

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