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CABARET FINITE-DIFFERENCE SCHEMES FOR THE ONE-DIMENSIONAL EULER EQUATIONS

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ABSTRACT

In the present paper we consider second order compact upwind schemes with a space split time derivative (CABARET) applied to one-dimensional compressible gas flows. As opposed to the conventional approach associated with incorporating adjacent space cells we use information from adjacent time layer to improve the solution accuracy. Taking the first order Roe scheme as the basis we develop a few higher (i.e. second within regions of smooth solutions) order accurate difference schemes. One of them (CABARET3) is formulated in a two-time-layer form, which makes it most simple and robust. Supersonic and subsonic shock-tube tests are used to compare the new schemes with several well-known second-order TVD schemes. In particular, it is shown that CABARET3 is notably more accurate than the standard second-order Roe scheme with MUSCL flux splitting.

1. INTRODUCTION

When solving hyperbolic equations such as the Euler equations numerically, one of the main problems that occurs is associated with the emergence of a wide range of scales in the solution. For the case of compressible gas flows, this range spans from weak acoustic pressure waves to large gradients propagating as shock waves.

To be robust in this context a numerical method should satisfy a variety of contrasting requirements. First, to be shock capturing the method should maintain integral properties and be conservative. On the other hand, such numerical schemes should account for a correct representation of genuine properties of the differential equations, such as characteristics, where possible, and have minimum dispersive (phase) and dissipative (amplitude) errors where the flow can be linearized. The failure of numerical methods to account for sharp gradients in the solution usually manifests itself in non-physical oscillations. The lack of accuracy, caused by a low order of approximation to differential equations, results in smearing out of solution amplitudes. Let us consider a simple linear convection equation.

$$\frac{\partial q}{\partial t} + \frac{\partial q}{\partial x} = 0. \tag{1.1}$$

In order to exclude non-physical oscillations in the solution, a finite-difference scheme should be monotonic. There is, however, a well-known theorem by S.K. Godunov [2] negating the existence of a liner difference scheme being both monotonic and higher than first order accurate. A traditional approach to avoiding this limitation consists of developing non-linear difference schemes – with non-linear fluxes.

For simplicity, let us consider a uniform finite-difference grid with a space step h and a time step τ. We will adopt the usual notation of indicating spatial nodes by subscripts and temporal nodes by superscripts. Traditionally one increases the approximation order for spatial and temporal terms separately – by constructing more accurate spatial fluxes first and then integrating the residual in time using a higher order method. A classical way of constructing second order accurate (in regions the solution is smooth) monotonic (Total Variation Diminishing) scheme is either to apply Harten's anti-diffusion idea using flux extrapolation from adjacent space cells [6]

$$\begin{aligned}
&\frac{u_i^{n+1} - u_i^n}{\tau} + \frac{u_i^n - u_{i-1}^n}{h}\Big[1 + \frac{\alpha_{i+1/2}}{2} \cdot \frac{u_{i+1}^n - u_i^n}{u_i^n - u_{i-1}^n} - \frac{\alpha_{i-1/2}}{2}\Big] = 0,\\
&\alpha_{i+1/2} = \alpha\Big(\frac{u_i^n - u_{i-1}^n}{u_{i+1}^n - u_i^n}\Big); \quad 0 \le \alpha_{i+1/2} \le 2 \\
&\text{and} \quad \alpha_{i+1/2} = 0 \quad \text{if} \quad \frac{u_{i+1}^n - u_i^n}{u_i^n - u_{i-1}^n} < 0,
\end{aligned} \tag{1.2}$$

or to exploit Van Leer's variable extrapolation method (MUSCL) using variables from adjacent space cells [10]

$$\begin{aligned}
&\frac{u_i^{n+1} - u_i^n}{\tau} + \frac{{u_{i+1/2}^n}^{left} - {u_{i-1/2}^n}^{right}}{h} = 0,\\
&{u_{i+1/2}^n}^{left} = u_i^n + \Big\langle\frac{\partial u}{\partial x}\Big\rangle_i^{left}\frac{h}{2} = u_i^n + \alpha\Big(\frac{u_i^n - u_{i-1}^n}{u_{i+1}^n - u_i^n}\Big) \cdot \frac{u_{i+1}^n - u_i^n}{2}.
\end{aligned} \tag{1.3}$$

Here α is a non-linear limiter function that deals with large gradients in solution.

Though in the linear case these two approaches result in identical finite-difference schemes, in the case of quasi-linear equations the approach based on MUSCL is known to be more robust [1]. Both these techniques have in common extrapolation from adjacent spatial cells. Hence their performance should be dependent on the quality of the space grid.

Alternatively, instead of using extra space cells one can use temporal information to gain more accuracy without having to extend the spatial stencil of the scheme. For instance, one can exploit a relatively unknown second-order linear CABARET scheme [3,4] (which is also known as Upwind Leapfrog Scheme [8]) regularized with non-linear correction [5]. The CABARET scheme can be presented as a combination of a conservation stage, updating the mid-cell variable,

$$\begin{aligned}
&\frac{\Psi_{i-1/2}^{n+1/2} - \Psi_{i-1/2}^{n-1/2}}{\tau} + \frac{u_i^n - u_{i-1}^n}{h} = 0, \\
&\Delta\Psi_{i-1/2}^n + \frac{\tau}{h}\delta u_{i-1/2}^n = 0,
\end{aligned} \tag{1.4}$$

where the nodal value u being referred to the cell walls and the mid-cell one Ψ being referred to the cell center, and a projection stage where the nodal variable is updated using an extrapolation rule. As part of the extrapolation stage, values at the nodes are corrected if found to be outside of the monotonicity range. The conservation property of the scheme holds because it is the mid-cell conservative variable which is marching in time.

$$\begin{aligned}
&\tilde{u}_i^{n+1} = 2\Psi_{i-1/2}^{n+1/2} - u_{i-1}^n, \\
&u_i^{n+1} = \tilde{u}_i^{n+1}, \\
&\text{if } \left(\tilde{u}_i^{n+1} > \max(u_i^n, u_{i-1}^n)\right) u_i^{n+1} = \max(u_i^n, u_{i-1}^n), \\
&\text{if } \left(\tilde{u}_i^{n+1} < \min(u_i^n, u_{i-1}^n)\right) u_i^{n+1} = \min(u_i^n, u_{i-1}^n).
\end{aligned} \tag{1.5}$$

The CABARET scheme with non-liner correction proved to be notably more robust in linear convection tests with coarse grids than standard TVD schemes. Our goal in the current paper is to develop CABARET counterparts for gas dynamics equations.

The paper is organized as follows. In section 2, we develop three new CABARET schemes for the Euler system. Numerical examples are provided in Section 3 for two shock tube problems where we are comparing the new schemes with a few classical second order TVD schemes commonly used in practice [7]. Finally, we end up with conclusions in Section 4.

2. COMPACT SCHEMES WITH SPACE-SPLIT TIME DERIVATIVES

2.1. CABARET and CABARET2 schemes

The CABARET scheme for the linear convection equation can be viewed as a superposition of first order upwind differences (Simple Upwind Scheme) and its conjugated scheme. It is natural to try to develop its gas dynamics counterpart using the same technique. Having chosen the first order Roe scheme as a gas-dynamic counterpart of Simple Upwind, one can construct CABARET schemes in a similar manner. A natural thing to do is to employ the CABARET formulae (1.4) devised for linear convection for each of the simple wave components in linearized wave equations. We refer the reader to [7] for the details of characteristic splitting techniques for the Euler system.

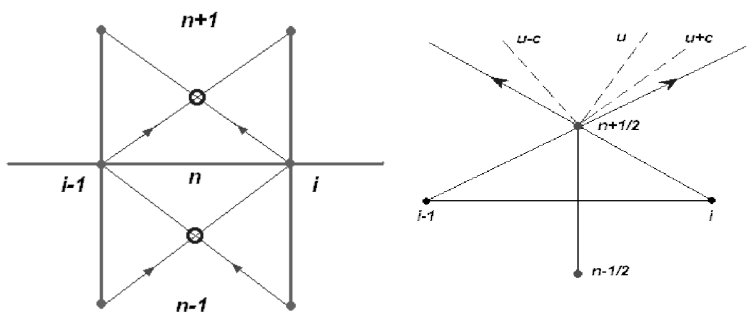

Figure 1. Numerical stencils for schemes CABARET2 and CABARET3.

Specifically, using the Roe matrix $\boldsymbol{L}^{Roe} \oplus \boldsymbol{U} = \boldsymbol{W}, \boldsymbol{U} = (\rho, \rho u, \rho E)$

$$\lambda_1 = \tilde{u},\ w_1 = \rho - \rho/\tilde{c}^2;\ \lambda_2 = \tilde{u} + \tilde{c},\ w_2 = u + \rho/(\tilde{\rho}\tilde{c});$$
$$\lambda_3 = \tilde{u} - \tilde{c},\ w_3 = u - \rho/(\tilde{\rho}\tilde{c}),$$
$$\tilde{\rho} = \frac{\sqrt{\rho_{i+1}\rho_i}}{\sqrt{\rho_{i+1}} + \sqrt{\rho_i}};\ \tilde{u} = \frac{u_{i+1}\sqrt{\rho_{i+1}} + u_i\sqrt{\rho_i}}{\sqrt{\rho_{i+1}} + \sqrt{\rho_i}};\ \tilde{H} = \frac{H_{i+1}\sqrt{\rho_{i+1}} + H_i\sqrt{\rho_i}}{\sqrt{\rho_{i+1}} + \sqrt{\rho_i}}.$$

we can split the Euler Jacobian

$$\boldsymbol{A}(\vec{U}) = \begin{pmatrix} 0 & 1 & 0 \\ \frac{\gamma - 3}{2}u^2 & (3-\gamma)u & \gamma - 1 \\ u^2(\gamma - 1) - \gamma u E & \gamma E - \frac{3}{2}(\gamma - 1)u^2 & \gamma u \end{pmatrix}$$

into characteristic variables $q = 1, 2, 3$ propagating with eigenvalues $\lambda_1, \lambda_2, \lambda_3$ (Fig.1)

$$\Delta^n \vec{\Psi}_q + \lambda_q \frac{\tau}{h} \delta_{i-1/2} \vec{u}_q = 0, \qquad (2.1)$$

where the 'conservative' central value is defined according to the direction of the 'wind'

$$\left(\vec{\Psi}_q\right)_{i-1/2}^{n+1/2} = \begin{cases} \left(\left(\vec{u}_q\right)_i^{n+1} + \left(\vec{u}_q\right)_{i-1}^n\right)/2 & \text{if } \lambda_q > 0, \\ \left(\left(\vec{u}_q\right)_{i-1}^{n+1} + \left(\vec{u}_q\right)_i^n\right)/2 & \text{if } \lambda_q < 0. \end{cases} \tag{2.2}$$

As characteristic splitting can be done using Roe formulae, the sum of all spatial contributions from different simple waves will be equal to the total flux difference

$$\Delta^n \vec{\Psi} = \sum_q \Delta^n \vec{\Psi}_q = -\frac{\tau}{h} \sum_q \lambda_q \delta_{i-1/2} \vec{u}_q = -\frac{\tau}{h}\left[F(\vec{U}_i^n) - F(\vec{U}_{i-1}^n)\right],$$

which should guarantee the conservation property for the scheme.

Depending on the eigenvalue sign in a cell one can compute the amplitudes of left and right running simple waves. According to the correction stage, then, the calculated variations should be checked to ensure that the maximum principle for each of them is satisfied. If not, the corresponding variations should be corrected (limited):

$$\begin{aligned} &\text{if} \quad \delta_{i-1/2}u \geq 0, \ -\delta_{i-1/2}u \leq \left(\Delta^{n+1/2}u_q\right)_i^{left} \leq 0; \\ &\qquad 0 \leq \left(\Delta^{n+1/2}u_q\right)_i^{right} \leq \delta_{i-1/2}u, \\ &\text{if} \quad \delta_{i-1/2}u < 0, \ 0 \leq \left(\Delta^{n+1/2}u_q\right)_i^{left} \leq -\delta_{i-1/2}u; \\ &\qquad \delta_{i-1/2}u \leq \left(\Delta^{n+1/2}u_q\right)_i^{right} \leq 0. \end{aligned}$$

Finally, the corrected contributions should be assembled at each node giving the total variation of conservative variable there.

Let us call the above algorithm CABARET1 scheme for the Euler equations. Though stable this scheme results in unsatisfactory dispersive qualities of the solution. The scheme, however, can be improved if the reconstruction of simple waves is modified. Specifically, during the discretization stage of the simple wave equations (2.1) one can add up regularizing dissipation [4] whose differential analogue is the addition of $-\varepsilon\frac{\partial^2}{\partial x \partial t}$ to the right-hand side of the convection equation. It was shown in [4] that this choice of regularizing dissipation greatly improves the dispersive properties of the CABARET scheme for linear convection equation. The introduction of this artificial dissipation amounts to the following modification of the interpolation rule (2.2):

$$\left(\vec{\Psi}_q\right)_{i-1/2}^{n+1/2} = \begin{cases} \left((1+\varepsilon)\left(\vec{u}_q\right)_i^{n+1} + (1-\varepsilon)\left(\vec{u}_q\right)_{i-1}^n\right)/2 & \text{if } \lambda_q > 0, \\ \left((1+\varepsilon)\left(\vec{u}_q\right)_{i-1}^{n+1} + (1-\varepsilon)\left(\vec{u}_q\right)_i^n\right)/2 & \text{if } \lambda_q < 0, \end{cases}$$

where $0 < \varepsilon < 1$. The formulae for simple waves amplitudes are

$$\text{if } \lambda_q > 0 \begin{cases} (1) \quad \left(\Delta^{n+1/2}\vec{u}_q\right)_i^{left} = \left(2\Delta^n \vec{\Psi}_q - (1-\varepsilon)(\Delta^{n-1/2}\vec{u}_q)_{i-1}\right)/(1+\varepsilon), \\ (3) \quad \left(\Delta^{n+1/2}\vec{u}_q\right)_i^{left} = \left(2\Delta^n \vec{\Psi}_q - (1-\varepsilon)(\Delta^{n-1/2}\vec{u}_q)_i\right)/(1+\varepsilon), \end{cases}$$

$$\left(\Delta^{n+1/2}\vec{u}_q\right)_{i-1}^{right} = 0;$$

$$\text{if } \lambda_q < 0 \begin{cases} (2) & \left(\Delta^{n+1/2}\vec{u}_q\right)_{i-1}^{right} = \frac{2\Delta^n\vec{\Psi}_q-(1-\varepsilon)(\Delta^{n-1/2}\vec{u}_q)_i}{1+\varepsilon}, \\ (4) & \left(\Delta^{n+1/2}\vec{u}_q\right)_{i-1}^{right} = \frac{2\Delta^n\vec{\Psi}_q-(1-\varepsilon)(\Delta^{n-1/2}\vec{u}_q)_{i-1}}{1+\varepsilon}, \end{cases}$$

$$\left(\Delta^{n+1/2}\vec{u}_q\right)_{i}^{left} = 0,$$

where cases (1) – (4) correspond to the eigenvalues signs $\left(\lambda_q\right)_{i-1/2}^n > 0$, $\left(\lambda_q\right)_{i-1/2}^{n-1} > 0$; $\left(\lambda_q\right)_{i-1/2}^n < 0, \left(\lambda_q\right)_{i-1/2}^{n-1} < 0$; $\left(\lambda_q\right)_{i-1/2}^n > 0, \left(\lambda_q\right)_{i-1/2}^{n-1} < 0$; $\left(\lambda_q\right)_{i-1/2}^n < 0, \left(\lambda_q\right)_{i-1/2}^{n-1} > 0$, respectively.

As in the CABARET1 scheme, the finding of left and right amplitudes of simple waves is followed by correction and assembling stages.

The modified scheme will be referred to as CABARET2 and can be viewed as a generalization of the CABARET1 ($\varepsilon = 0$) and first order Roe ($\varepsilon = 1$) schemes.

2.2. CABARET3 scheme

The presence of tunable parameters is not a pleasant thing for a method. A conceptual drawback of CABARET1 and CABARET2 is that they are both based on simple wave reconstruction using simple waves from different time layers which lacks physical foundation.

A successful approach, which avoids this, lies in the combination of conservative step performed using conservative variables, i.e. density, momentum and energy, with a characteristic splitting applied after that.

Specifically, the first conservative sub-step advances the conservative central variables in time

$$\frac{\vec{\Psi}_{i-1/2}^{n+1/2} - \vec{\Psi}_{i-1/2}^{n-1/2}}{\tau} + \frac{Flux(\vec{U}_i^n) - Flux(\vec{U}_{i-1}^n)}{h} = 0. \tag{2.3}$$

This sub-step makes the scheme unconditionally conservative regardless of the later splitting. After the conservative sub-step each cell has a set of vectors of conservative variables defined at the cell center and conservative variables defined at the left and right nodal points.

At the second sub-step these vectors are multiplied by the left Roe characteristic matrix

$$\begin{pmatrix} 1-\frac{\gamma-1}{2}u^2/c^2 & (\gamma-1)u/c^2 & -(\gamma-1)/c^2 \\ \left(\frac{\gamma-1}{2}u^2-uc\right)/(\rho c) & \left(c-(\gamma-1)u\right)/(\rho c) & (\gamma-1)/(\rho c) \\ -\left(\frac{\gamma-1}{2}u^2+uc\right)/(\rho c) & \left(c+(\gamma-1)u\right)/(\rho c) & -(\gamma-1)/(\rho c) \end{pmatrix}$$

and transformed into the vectors of Riemann variables:

$$\begin{aligned} &\vec{w}_{i-1/2}^{n+1/2} = \boldsymbol{L}^{Roe} \oplus \vec{\Psi}_{i-1/2}^{n+1/2}, \\ &\vec{w}_i^n = \boldsymbol{L}^{Roe} \oplus \vec{U}_i^n, \\ &\vec{w}_{i-1}^n = \boldsymbol{L}^{Roe} \oplus \vec{U}_{i-1}^n. \end{aligned} \tag{2.4}$$

This is a vector analogue of the scalar formulae (1.4), however, in the vector case the characteristic variables in the mid-cell $(n+1/2)$ come already resolved, being obtained from $\Psi^{n+1/2}$.

To update values at nodal points, then, one can use the CABARET projection formula together with correction (1.5) for each characteristic variable

$$\begin{aligned} &\left(\tilde{w}_i^{n+1}\right)^{left} = 2w_{i-1/2}^{n+1/2} - w_{i-1}^n; \ \left(\tilde{w}_{i-1}^{n+1}\right)^{right} = w_{i-1}^n \quad \text{if } \lambda_{i-1/2} > 0, \\ &\left(\tilde{w}_{i-1}^{n+1}\right)^{right} = 2w_{i-1/2}^{n+1/2} - w_i^n; \ \left(\tilde{w}_i^{n+1}\right)^{left} = w_i^n \quad \text{if } \lambda_{i-1/2} < 0, \\ &\text{if } \left(\tilde{w}_i^{n+1} > \max(w_i^n, w_{i-1}^n)\right) w_i^{n+1} = \max(w_i^n, w_{i-1}^n), \\ &\text{if } \left(\tilde{w}_i^{n+1} < \min(w_i^n, w_{i-1}^n)\right) w_i^{n+1} = \min(w_i^n, w_{i-1}^n). \end{aligned}$$

The corrected variations of Riemann variables $dw = w^{n+1} - w^n$ are assembled into the variations of conservative variables by multiplying by the right characteristic matrix

$$\begin{pmatrix} 1 & \rho/(2c) & -\rho/(2c) \\ u & \rho(u+c)/(2c) & -\rho(u-c)/(2c) \\ \frac{1}{2}u^2 & \rho u(H+uc)/(2c) & -\rho u(H-uc)/(2c) \end{pmatrix}.$$

Finally, by adding up left and right conservative variations, the total nodal variation of conservative variable is obtained

$$\begin{aligned} &\left(d\vec{U}_i^{n+1}\right)^{left} = \boldsymbol{R}^{Roe} \oplus \left(dw_i^{n+1}\right)^{left}; \\ &\left(d\vec{U}_i^{n+1}\right)^{right} = \boldsymbol{R}^{Roe} \oplus \left(dw_i^{n+1}\right)^{right}; \\ &d\vec{U}_i^{n+1} = \left(d\vec{U}_i^{n+1}\right)^{left} + \left(d\vec{U}_i^{n+1}\right)^{right}. \end{aligned}$$

Due to the properties of the Roe Jacobian sonic points $u \approx c$ c are to be treated separately. A quite robust recipe consists of using the first order upwind approximation with modified eigenvalue (P.Roe) in the vicinity of sonic points. For $\lambda_{1-1/2} > 0$ this amounts to letting $\left(dw_i^{n+1}\right)^{left} = -\varepsilon(w_i^n - w_{i-1}^n)\tau/h$; $\left(dw_{i-1}^{n+1}\right)^{right} = 0$, where $\varepsilon = \max\{0, (\lambda_{i+1/2} - \lambda_i), (\lambda_{i+1} - \lambda_{i+1/2})\}$.

At the initial time step one can use the first order Roe scheme, which follows by replacing the previous mid-cell value by the arithmetic mean of the values at nodal walls

$$\frac{\vec{\Psi}_{i-1/2}^{n+1/2} - 0.5(\vec{U}_i^n + \vec{U}_{i-1}^n)}{0.5\tau} + \frac{Flux(\vec{U}_i^n) - Flux(\vec{U}_{i-1}^n)}{h} = 0.$$

This brings us to a simple robust, tuning-free scheme which will be referred to as CABARET3. This scheme is conservative, second order in the regions of smooth solutions and it prevents the appearance of non-physical oscillations.

3. NUMERICAL EXAMPLES

Two shock tube problems by Sod have been chosen as test cases for the numerical methods developed. These are:

1. Subsonic test

$$(\rho, u, p)^{left} = (1, 0, 1.e+5); \ (\rho, u, p)^{right} = (0.125, 0, 1.e+4).$$

2. Supersonic test

$$(\rho, u, p)^{left} = (1, 0, 1.e+5); \ (\rho, u, p)^{right} = (0.01, 0, 1.e+3).$$

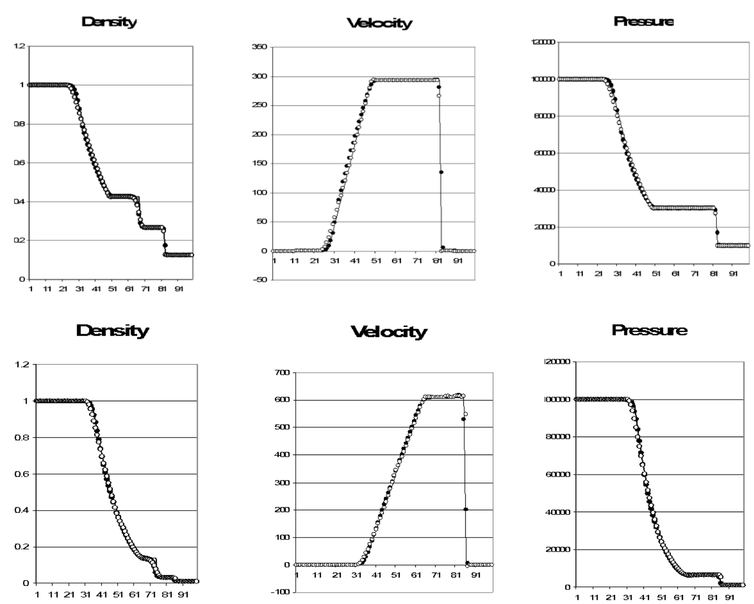

Figure 2. Shock-tube problems – comparison between MUSCL with Min-Mod versus CABARET2 at the subsonic test (up) supersonic test (low).

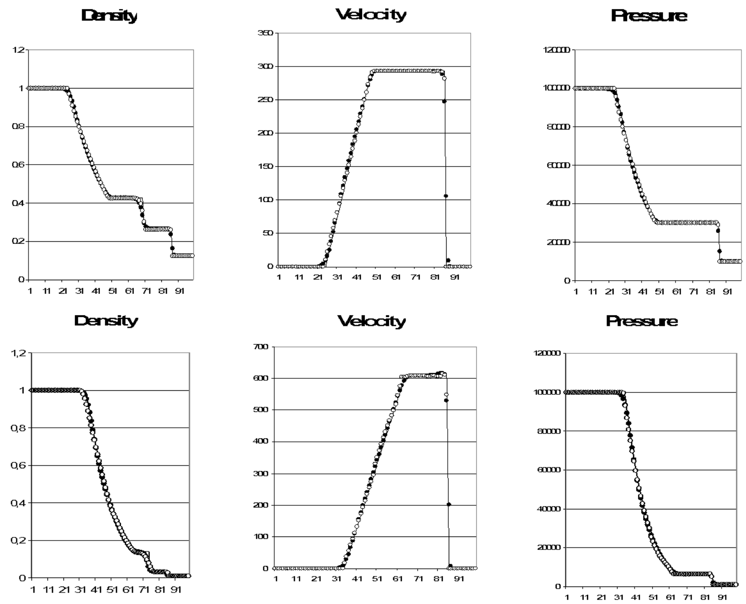

Figure 3. Shock-tube problems – comparison between MUSCL with MinMod versus CABARET3 at the subsonic test (up) supersonic test (low).

Several traditional second-order TVD schemes [7] based on characteristic splitting have been tested

- Using flux extrapolation: second order Roe upwind scheme (four types of upwind biased limiters: Van Leer's, MinMod, Superbee, and a third order accurate limiter from [11], referred to as MSU here), Lax-Wendroff scheme, and Yee's version of LW (three types of symmetrical limiters based on MinMod);

- Using variable extrapolation with Roe scheme: MUSCL with four types of upwind biased limiters; and
- CABARET, CABARET2 and CABARET3 schemes

The main results of the comparison among traditional TVD schemes agree with [7]. The upwind schemes (Roe) tend to be more accurate than the centered difference schemes (Lax-Wendroff and Yee). On the other hand, the performance of centered schemes depends slightly less on CFL number. Among the limiters the MinMod limiter is seen to be the most robust in terms of clean work within a wide CFL region. Superbee and MSU limiters are more compressive but their performance is greatly affected by CFL number.

The variable extrapolation seems to be superior to the flux extrapolation, which have been compared on the example of the first order Roe scheme, though there is no striking difference in performance between flux and variable extrapolation noticed.

Fig.2 shows the comparison of CABARET2 with $\varepsilon = 0.3$ versus Roe MUSCL MinMod. It can be seen that CABARET2 scheme gives a comparable quality of results, showing good shock wave resolution but slightly smearing the corners of the expansion fan.

Fig.3 demonstrates results of the CABARET3 scheme in comparison to Roe MUSCL MinMod. It can be seen that CABARET3 is not only very good at shock/contact wave resolution but also at capturing the edges of the expansion fan.

Being compact, a CABARET scheme should be less sensitive to the space grid quality than standard, not as compact, schemes. To explore this, CABARET3 and Roe MUSCL MinMod were tested on a very irregular grid whose cell width differed by an order of magnitude $h_{\max}/h_{\min} = 20$. As demonstrated in Fig.4 CABARET3 is notably more accurate with this grid as well.

4. CONCLUSIONS

In the paper a few compact second order accurate conservative schemes of CABARET-type are developed for the one-dimensional Euler equations. These schemes can be viewed as generalizations of the characteristic Roe scheme by incorporating an additional time layer. Importance of two-time layer characteristic splitting for CABARET schemes is emphasized. Several well-known second order TVD schemes are considered and the second order Roe scheme with MUSCL flux splitting using MinMod limiter is chosen as the most robust representative of 'industry standard' methods. In comparison to the 'the industry standard' scheme, CABARET3 is shown to be more accurate both in resolving shock/contact waves and capturing the edges of expansion fans.

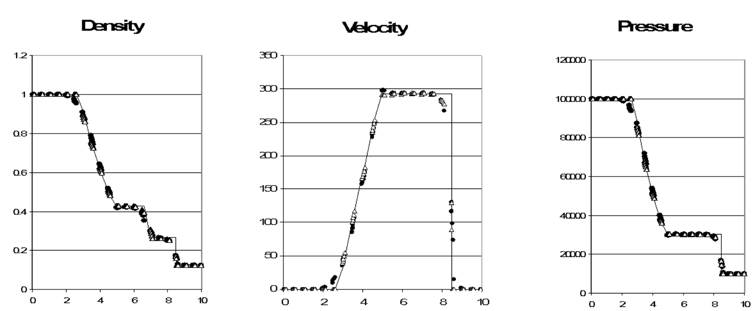

Figure 4. Shock-tube problems – comparison between MUSCL with MinMod versus CABARET3 on highly irregular mesh with $h_{\max}/h_{\min} = 20$ at the subsonic test.

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REFERENCES

[1] W.K. Anderson, J.L. Thomas, B. Van Leer. Comparison of finite volume Flux vector splittings for the Euler equations. *AIAA Journ.*, **24** 1986, 1453i – 60.

[2] S.K. Godunov. A difference scheme for numerical computation of discontinuous solutions of equations of fluid dynamics. *Math.Sb.*, **47** (89), 1959, 271 – 306.

[3] V.M. Goloviznin, A.A. Samarskii. A finite-difference approximation of convective transport with a space-split time derivative. *Journ. Of. Math. Modelling*, **10** (1), 1998, 86 – 100.(in Russian)

[4] V.M. Goloviznin, A.A. Samarskii. On several properties of the CABARET scheme. *Journ. Of. Math. Modelling*, **10** (1), 1998, 101 – 116.(in Russian)

[5] V.M. Goloviznin, S.A. Karabasov. Article in journal. Non-linear correction of Cabaret scheme. *Journ. of Math. Modelling*, **10** (12), 1998, 107 – 123.(in Russian)

[6] A. Harten. High resolution schemes for hyperbolic conservation laws. *J.Comput. Phys.*, **49** 1983, 357 – 393.

[7] C. Hirsh. *Numerical computation of internal and external flows, V2: Computational Methods for Inviscid and Viscous Flows.* John Wiley & Sons Ltd, 1990.

[8] A. Iserles. Generalized leapfrog methods. *IMA Journal of Numerical Analysis*, **6** 1986.

[9] P.L. Roe. Characteristic based schemes for the Euler equations. *Annual. Rev. Fluid. Mech.*, **18** 1986, 337 – 365.

[10] B. Van Leer. Towards the ultimate conservative difference scheme. II. Monotonicity and conservation combined in a second order scheme. *Journal of Comp. Phys.*, **14** 1974, 361 – 70.

[11] K.V. Vyaznikov, V.F. Tishkin, A.P. Favorski, M.Yu. Shashkov. Quasi-monotone finite-difference schemes of higher accuracy. *Preprint IPM AN SSSR*, (36), 1987.(Russian)

Vienmačių Eulerio lygčių sprendimas CABARET3 baigtinių-skirtumų schemomis

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Šiame straipsnyje nagrinėjamos kompaktiškos antrosios tikslumų eilės baigtinių skirtumų schemos, kuriose panaudota speciali išvestinių aproksimacija. Sprendžiamas vienmatis spūdžių dujų judėjimo uždavinys. Darbe pasiūlytas didesnio tikslumo schemų konstravimo metodas, kuriame išnaudojama informacija apie sprendinį iš žemesnio laiko sluoksnio. CABARET3 schema yra dvisluoksnė, todėl jos realizavimo algoritmas yra ekonomiškas. Pateikiami skaičiavimo eksperimento rezultatai, kurie patvirtina, kad CABARET3 schema yra tikslesnė už antrosios tikslumo eilės Roe schemą, naudojančią MUSCL srauto išskaidymą.