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NATURAL CONVECTION FLOW ON A SPHERE THROUGH POROUS MEDIUM IN PRESENCE OF HEAT SOURCE/SINK NEAR A STAGNATION POINT

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Abstract. Steady natural convection boundary-layer flow in the neighbourhood of lower stagnation point of a heated sphere embedded in a saturated porous medium in presence of heat source/sink is considered. The dimensionless governing equations for this investigation are solved numerically by shooting method. Then attention is focused on finding the effects of porosity parameter, heat source/sink parameter and the Prandtl number on velocity and temperature distributions. With the increase of permeability parameter of the porous medium, the fluid velocity decreases but the temperature increases at a particular point of the surface. Due to increasing values of heat source/sink parameter, fluid velocity is found to increase. It is noticed that the velocity boundary layer is suppressed with the increasing Prandtl number and the temperature decreases in this case. Prandtl number has an important effect in increasing the rate of heat transfer from the surface of the sphere.

Key words: Steady flow, Natural convection, Porous medium, Sphere, Stagnation point, Heat source/sink.

1 Introduction

Convective heat transfer within fluid saturated porous media has attracted considerable attention because of it's ever increasing applications in geophysics, oil recovery techniques, thermal insulation engineering, packed-bed catalytic reactors and heat storage beds. A wide variety of these applications involving convective transport phenomena is cited by numerous authors. Comprehensive studies on convective flow in porous media were performed by Vafai (2000) [14], Pop and Ingham (2001) [12], Ingham and Pop (2002) [9] and others.

Most of the studies refer to bodies of relatively simple geometry, such as, cylinders, spheres, flat plates (see, Chen and Mucoglu (1977) [3], Cheng (1982)

[4]). Free convection from a uniformly heated sphere, in which the fluid motion is generated due to buoyancy forces, has received much more attention recently. Potter and Riley (1980)[13] studied the free-convective flow from a heated sphere at large Grashof number. Free convection at an axisymmetric stagnation point was considered by Amin and Riely (1996) [1]. Very recently, Hossain *et al.* (2004) [7] studied the conjugate effect of heat and mass transfer in natural convection flow from an isothermal sphere with chemical reaction.

Study of flow through porous media finds its applications in a broad spectrum of disciplines covering chemical engineering to geophysics. Fluid flow and heat transfer through porous media towards the surface of a body have an important bearing on several technological processes. The working fluid heat generation (source) or absorption (sink) effects are important in certain porous media applications such as those involving heat removal from nuclear fuel debris, underground disposal of radioactive waste materials, storage of food stuffs and exothermic or endothermic chemical reactions and dissociating fluids in packed-bed reactors. Mendez and Trevino (2000) [10] studied the effects of the conjugate conduction-natural convection heat transfer along a thin vertical plate with non-uniform heat generation. Hossain *et al.* (2004) [8] discussed the problem of natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat source/sink. Unsteady MHD convective heat and mass transfer past a semi infinite vertical permeable moving plate with heat absorption, was considered by Chamkha (2004) [2]. Molla *et al.* (2005) [11] investigated the effects of magneto-hydrodynamic natural convection flow on a sphere in presence of heat source.

Few works are available in literature on the subject of natural convection flow in porous media. Heat generation/absorption effect on free convection flow from an isothermal sphere embedded in a saturated porous medium near a stagnation point had not yet been addressed and the present work considers this topic. The objective of this paper is to focus on the above issue by considering natural convection boundary-layer flow from a sphere embedded in a porous medium near the lower stagnation point in presence of heat source/sink. By adopting appropriate transformations, the governing equations are reduced to locally non-similar partial differential equations. The set of equations reduce to ordinary differential equations at the lower stagnation point ($x = 0$) of the sphere. These equations are solved numerically by shooting method. Velocity and temperature distributions for a selection of parameters such as permeability, heat source/sink and for the Prandtl number are obtained and discussed in results and discussions section.

2 Equations of Motion

We consider the two-dimensional steady natural convection flow of an incompressible viscous liquid over a sphere of radius a embedded in a porous medium. The coordinates $\bar{x}$ and $\bar{y}$ measure the distance along the surface of the sphere from the stagnation point and the distance normal to the surface of the sphere respectively. A sketch of the physical problem and the coordinate system are shown in Fig.1.

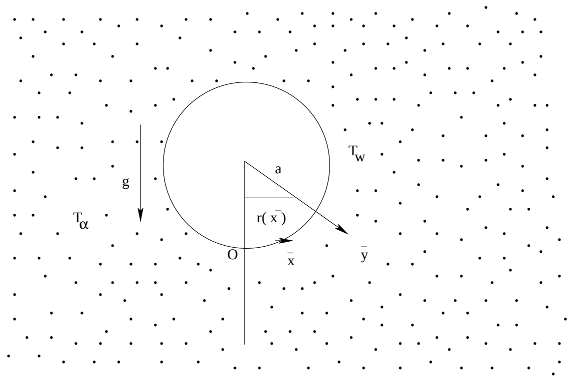


Figure 1. Sketch of the physical model and the coordinate system.

The governing boundary layer equations with the application of Darcy's law [Vafai (2000) [14]] and under the Boussinesq approximation in the usual notation are given by

$$\frac{\partial(\bar{r}\bar{u})}{\partial\bar{x}} + \frac{\partial(\bar{r}\bar{v})}{\partial\bar{y}} = 0, \tag{2.1}$$

$$\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}} = g\beta(T - T_\infty)\sin(\frac{\bar{x}}{a}) + \nu\frac{\partial^2\bar{u}}{\partial\bar{y}^2} - \frac{\nu}{k}\bar{u}.$$

Here $\bar{r}(\bar{x}) = a\sin(\frac{\bar{x}}{a})$ is the radial distance from the symmetrical axis of the sphere, $\bar{u}$ and $\bar{v}$ are the components of velocity respectively in the $\bar{x}$ and $\bar{y}$ directions, k is the permeability of the porous medium, μ is the coefficient of fluid viscosity, ρ is the fluid density, $\nu = \mu/\rho$ is the kinematic viscosity, β is the coefficient of thermal expansion, g is the acceleration due to gravity.

By using the boundary layer approximations and neglecting viscous dissipation, the equation for temperature is given by

$$\bar{u}\frac{\partial T}{\partial\bar{x}} + \bar{v}\frac{\partial T}{\partial\bar{y}} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial\bar{y}^2} + \frac{Q_0}{\rho c_p}(T - T_\infty), \tag{2.2}$$

where T is the temperature, T_∞ is the free stream temperature, κ is the thermal conductivity, c_p is the specific heat at constant pressure, Q_0 is the heat generation ($Q_0 > 0$) or absorption ($Q_0 < 0$) constant.

The appropriate boundary conditions for the above problem are given by

$$\bar{u} = 0,\ \ \bar{v} = 0,\ \ T = T_w \quad at \quad \bar{y} = 0,$$

$$\bar{u} \to 0, T \to T_\infty \quad as \quad \bar{y} \to \infty.$$

Here T_w is the uniform surface temperature of the sphere. Constants T_w and T_∞ satisfy the inequality $T_w > T_\infty$.

3 Method of Solution

Let us introduce the following dimensionless variables (Molla *et al.* (2005) [11])

$$x = \frac{\bar{x}}{a}, \;\; y = (Gr)^{\frac{1}{4}}\frac{\bar{y}}{a}, \;\; u = \frac{a}{\nu}(Gr)^{-\frac{1}{2}}\bar{u}, \;\; v = \frac{a}{\nu}(Gr)^{-\frac{1}{4}}\bar{v},$$
$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \;\; Gr = \frac{g\beta(T_w - T_\infty)a^3}{\nu^2},$$

where Gr is the Grashof number and θ is the non-dimensional temperature. Then equations (2.1)–(2.2) are transformed to

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0,$$
$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \theta \sin x + \frac{\partial^2 u}{\partial y^2} - \frac{a^2}{kGr^{\frac{1}{2}}}u, \tag{3.1}$$
$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} + \frac{Q_0 a^2}{\rho c_p \nu Gr^{\frac{1}{2}}}\theta. \tag{3.2}$$

The boundary conditions take the following form:

$$u = 0, \;\; v = 0, \;\; \theta = 1 \quad at \quad y = 0,$$
$$u \to 0, \;\; \theta \to 0 \quad as \quad y \to \infty.$$

Introducing the non dimensional stream function $\psi = xr(x)f(x, y)$, where

$$u = \frac{1}{r}\frac{\partial \psi}{\partial y}, \quad v = -\frac{1}{r}\frac{\partial \psi}{\partial x}$$

and putting them into equations (3.1) and (3.2) we get

$$\frac{\partial^3 f}{\partial y^3} + \left(1 + \frac{x\cos x}{\sin x}\right)f\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial f}{\partial y}\right)^2 + \theta\frac{\sin x}{x} - k_1\frac{\partial f}{\partial y} = x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y \partial x} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right),$$
$$\frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} + \left(1 + \frac{x\cos x}{\sin x}\right)f\frac{\partial \theta}{\partial y} + \lambda\theta = x\left(\frac{\partial f}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial \theta}{\partial y}\right),$$

where $k_1 = a^2/(kGr^{\frac{1}{2}})$ is the permeability parameter of the porous medium (Cortell (2005) [6]) and $\lambda = Q_0 a^2/(\mu c_p Gr^{\frac{1}{2}})$ is the heat source/sink parameter.

At the lower stagnation point ($x = 0$), the above equations reduce to the following ordinary differential equations:

$$f''' + 2ff'' - f'^2 + \theta - k_1 f' = 0, \tag{3.3}$$
$$2f\theta' + \lambda\theta + \frac{1}{Pr}\theta'' = 0. \tag{3.4}$$

The boundary conditions now become

$$f' = 0, \;\; f = 0 \quad \text{and} \quad \theta = 1 \quad \text{at} \quad y = 0,$$
$$f' \to 0, \;\; \theta \to 0 \quad \text{as} \quad y \to \infty.$$

4 Numerical Method for Solution

The equations (3.3) and (3.4) along with boundary conditions are solved by converting this boundary value problem to an initial value problem (Conte and Boor (1972) [5]). We set

$$\begin{cases} f' = z, \quad z' = p, \quad p' = z^2 - 2fp - \theta + k_1 z, \\ \theta' = q, \quad q' = -Pr(2fq + \lambda\theta) \end{cases} \tag{4.1}$$

with the boundary conditions

$$f(0) = 0, \quad f'(0) = 0, \quad \theta(0) = 1.$$

In order to integrate system (4.1) as an initial value problem we require a value for $p(0)$ (i.e. for $f''(0)$) and $q(0)$ (i.e. for $\theta'(0)$) but no such values are given at the boundary. Using shooting method suitable guess values for $f''(0)$ and $\theta'(0)$ are chosen and then integration is carried out. We compare the calculated values for f' and θ at $y = 6$ (say) with the given boundary conditions $f'(6) = 0$ and $\theta(6) = 0$ and adjust the estimated values, $f''(0)$ and $\theta'(0)$, to give a better approximation for the solution.

5 Results and Discussions

Computation using the described numerical scheme has been carried out for various values of the parameters such as heat source/sink parameter λ, permeability parameter k_1 and Prandtl number Pr. For illustrations of the results, numerical values are plotted in the Fig. 2–7. Our results (with $k_1 = 0$) agree well with those of Molla *et al.* (2005) [11] (wih $x = 0$, $M = 0$).

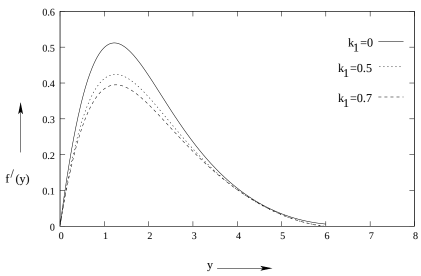


Figure 2. Variation of velocity profiles with y for various values of k_1 when $\lambda = 0$ and Pr=0.1.

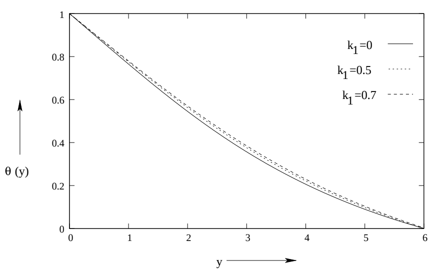
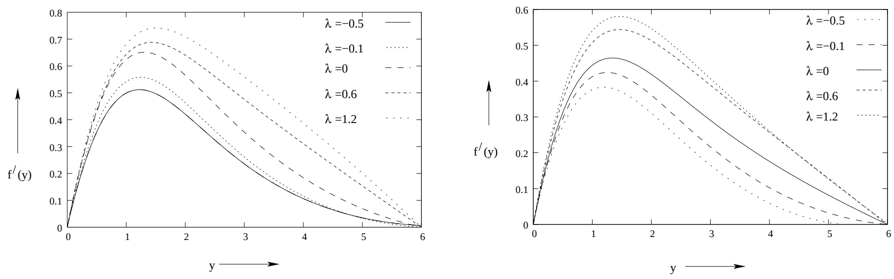


Figure 3. Variation of temperature with y for various values of k_1 when $\lambda = 0$ and Pr=0.1.

Figures 2 and 3 present horizontal velocity profile $f'(y)$ and temperature profile $\theta(y)$ for different values of the permeability parameter k_1 of the porous medium ($k_1 = 0.0, 0.5, 0.7$), when the Prandtl number $Pr = 0.1$ in the absence of heat source/sink. It is found that the horizontal velocity $f'(y)$ decreases with the increase of k_1 but the temperature increases with the increase of k_1. The increase of permeability parameter k_1 leads to increase the skin-friction. The permeability parameter k_1 introduces additional shear stress on the boundary whereas the thermal boundary layer thickness becomes thinner with the decreasing permeability parameter k_1.

Figures 4(a),(b) demonstrate the effects of heat source/sink on the velocity profiles in the absence ($k_1 = 0$) and presence ($k_1 = 0.5$) of porous media respectively. It is noticed that horizontal velocity increases with the increase of heat source/sink parameter λ (=-0.5, -0.1, 0, 0.6, 1.2) ($Pr = 0.1$) in both cases. Near the surface of the sphere, velocity increases significantly and then decreases slowly and finally vanishes at $y = 6$.



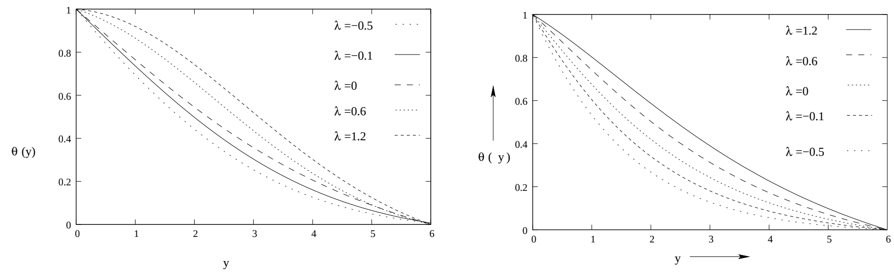
(a) (b)

Figure 4. Variation of velocity profiles with y for several values of λ when a)$k_1 = 0$ and $Pr = 0.1$, b) $k_1 = 0.5$ and $Pr = 0.1$.

Figures 5(a), (b) display the nature of the temperature field for different values of the heat source/sink parameter λ (=-0.5, -0.1, 0, 0.6, 1.2) ($Pr = 0.1$) in the absence ($k_1 = 0$) and in the presence ($k_1 = 0.5$) of porous media respectively. In this case, the temperature field increases with the increase of heat source parameter. This feature prevails up to certain heights and then the process is slowed down and at a far distance from the surface of the sphere (at $y = 6$), temperature vanishes. On the other hand, the temperature field increases with the decrease in the amount of heat absorption. Again, far away from the surface of the sphere, such feature is smeared out. The presence of heat absorption ($\lambda < 0$) creates a layer of cold fluid adjacent to the heated surface of the sphere and so the heat transfer rate from the surface of the sphere increases in this case. Fig.6 and Fig.7 exhibit the effects of Prandtl number ($Pr = 0.01, 0.05, 0.1$) on the velocity and temperature fields when $k_1 = 0.5, \lambda = 0$. Both the velocity and the temperature are found to decrease with the increase of the value of the Prandtl number. The thermal boundary layer thickness decreases due to increase of Prandtl number Pr.

6 Conclusion

Natural convection flow and heat transfer near a lower stagnation point of a sphere immersed in a saturated porous medium with internal heat generation/absorption is investigated by presenting similarity solutions. The effect of permeability parameter is found to decrease the horizontal velocity and to



(a) (b)

Figure 5. Variation of temperature with y for several values of λ when a) $k_1 = 0$ and $Pr = 0.1$, b) $k_1 = 0.5$ and $Pr = 0.1$.

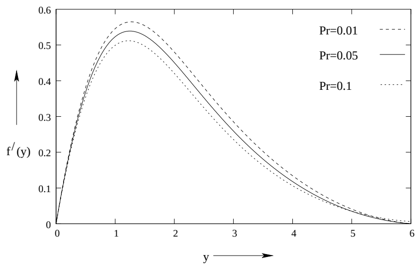


Figure 6. Variation of velocity profiles with y for several values of Pr when a)$k_1 = 0.5$ and $\lambda = 0$.

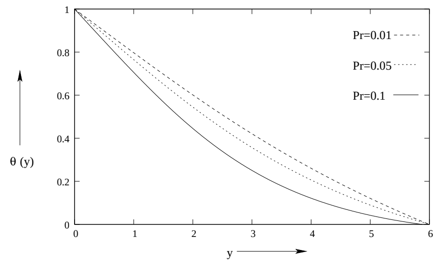


Figure 7. Variation of temperature with y for several values of Pr when a) $k_1 = 0.5$, $\lambda = 0$.

increase the temperature. In presence of heat generation (absorption) parameter, both the temperature and the fluid velocity increase (decrease). With the increasing Prandtl number, fluid velocity and temperature are found to decrease.
It is hoped that the physics of flow over the surface of the sphere can be utilized as the basis for many scientific and engineering applications and for studying more complex problems of porous media. The results of the problem are also of great interest in modern technology.

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