

MODELLING OF THE PROCESS OF HEATING GLASS FABRIC

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During the industrial process of manufacturing the insulated technical fabric from the glass fibre there remains considerable quantity of oil on its surface, as the oil is necessary for forming the required glass fibres. A special furnace is used for removing oil from fabric, in which the fabric, moving inside the furnace, is heating up to the oil burnout temperature by means of built-in heaters. The resultant high fabric temperatures are known to influence the intrinsic structure of the material and may cause the tensile strength of the glass fabric to decrease.

That is why it is necessary to consider the problem connected with the processes of heating fabric and oil burnout. The sequential investigation carried out separately makes it possible to determine some of the unknown parameters of the process and to compare them with the experimental data. Besides, it allows one to justify the assumptions used in the modelling.

1. The formulation of the heat problem. The temperature distribution T in the fabric is described by the basic two-dimensional equation:

$$\rho c_p \frac{\partial T}{\partial t} = \lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - v \rho c_p \frac{\partial T}{\partial x}, \quad 0 < x < D, \quad 0 < y < \delta, \quad (1)$$

where D is the length of the furnace and δ is the thickness of the fabric. The fabric is moving along the X -axis with the velocity v . ρ , c_p , λ are the density, heat capacity and heat conductivity of the fabric material. For the fabric surface two kinds of boundary conditions are considered:

I. The simple Stefan-Boltzman law

$$\text{for } y=0: \quad -\lambda \frac{\partial T}{\partial y} = \epsilon_L \sigma (T_N^4 - T^4) - \alpha (T - T_g) \quad (2)$$

$$\text{for } y=\delta: \quad \lambda \frac{\partial T}{\partial y} = \epsilon_L \sigma (T_k^4 - T^4) - \alpha (T - T_g) \quad (3)$$

II. With the reflected heat fluxes we therefore obtain

$$\text{for } y=0: \quad -\lambda \frac{\partial T}{\partial y} = -\frac{\epsilon_L}{1-\epsilon_L} (\sigma T^4 - q_0) - \alpha (T - T_g) \quad (4)$$

for $y=\delta$ is the condition (3), q_0 and q_L are the reflected heat fluxes [3]:

$$\begin{cases} q_0(x,t) - (1-\epsilon_0) \int_0^D q_L(\xi_1,t) \frac{a^2}{2[(\xi_1-x)^2+a^2]^{3/2}} d\xi_1 = \epsilon_0 \sigma T_N^4(x,t) \\ q_L(\xi,t) - (1-\epsilon_L) \int_0^D q_0(x_1,t) \frac{a^2}{2[(x_1-\xi)^2+a^2]^{3/2}} dx_1 = \epsilon_L \sigma T^4(\xi,t) \end{cases} \quad (5)$$

$$0 < x < D, \quad 0 < \xi < D, \quad 0 < x_1 < D, \quad 0 < \xi_1 < D$$

$$\text{For } x=0: \quad T = T_0, \quad \text{for } x=D: \quad \frac{\partial T}{\partial x} = 0$$

As far as the thickness of the fabric (δ) is an order of magnitude less than its other characteristic sizes, we can assume that the fabric temperature is constant in thickness. Having carried out averaging on the Y -axis with regard to the corresponding boundary conditions we obtain the following final equation for the fabric:

$$\rho c_p \frac{\partial u}{\partial t} = \lambda \frac{\partial^2 u}{\partial x^2} - \nu \rho c_p \frac{\partial u}{\partial x} - \frac{\epsilon_L \sigma (2 - \epsilon_L)}{\delta (1 - \epsilon_L)} u^4 + \frac{\epsilon_L \sigma}{\delta} T_k^4 + \frac{\epsilon_L}{\delta (1 - \epsilon_L)} q_0 - \frac{2\alpha}{\delta} (u - T_g), \quad (6)$$

$$0 < x < D$$

The numerical results for the given model were compared with the experimental data to allow parameter ranges to be determined for the emissivities ϵ_D and ϵ_L and the heat transfer coefficient α .

Besides, in addition to comparison of the calculation results according to various models we have shown the necessity of taking into account heat transfer radiation between the fabric and the heater in the form of (4), i.e. with regard to the reflected radiation.

For the concrete calculations according to the above model several additions have been made.

1. The heater temperature is not a constant. Since at the beginning of the furnace as well as at its end there is a row of bricks 0.3 m wide, whose temperature is below that of the heater (especially at the furnace inlet), the temperature distribution on the furnace initial part of 0.44 m (including bricks) was taken from 70°C , at the first point to 800°C . At the last point at the chamber outlet (as well as along the bricks)

the temperature was taken 800°C .

2. Apart from that, at the furnace outlet the fabric enters the additional chamber, 0.6 m long, whose temperature varies lineary from 800°C to 100°C in the upper heater.

In Fig. 1, there is shown the temperature distribution along the fabric inside the furnace (up to 1.76 m), inside the chamber (up to 2.36 m), and after the outlet. In this and following figures, the continuous line depicts the temperature distribution on the fabric according to model (1)–(3) with radiation by the Stefan–Boltzman law; the line with asterisks corresponds to model (1)–(5) with regard to the reflected heat fluxes.

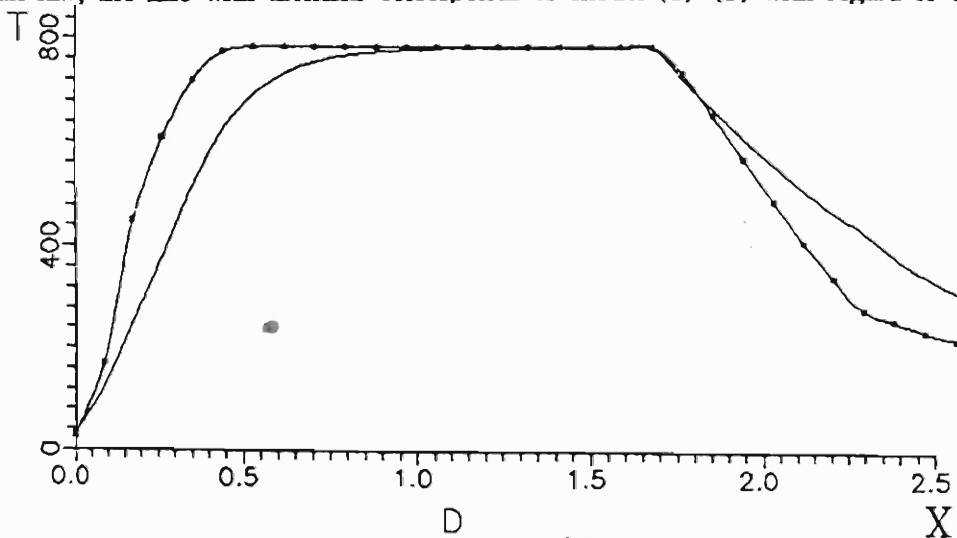


Fig. 1

In Fig. 2, there are given the temperature gradients ($^\circ\text{C}/\text{m}$) along the fabric under the above conditions.

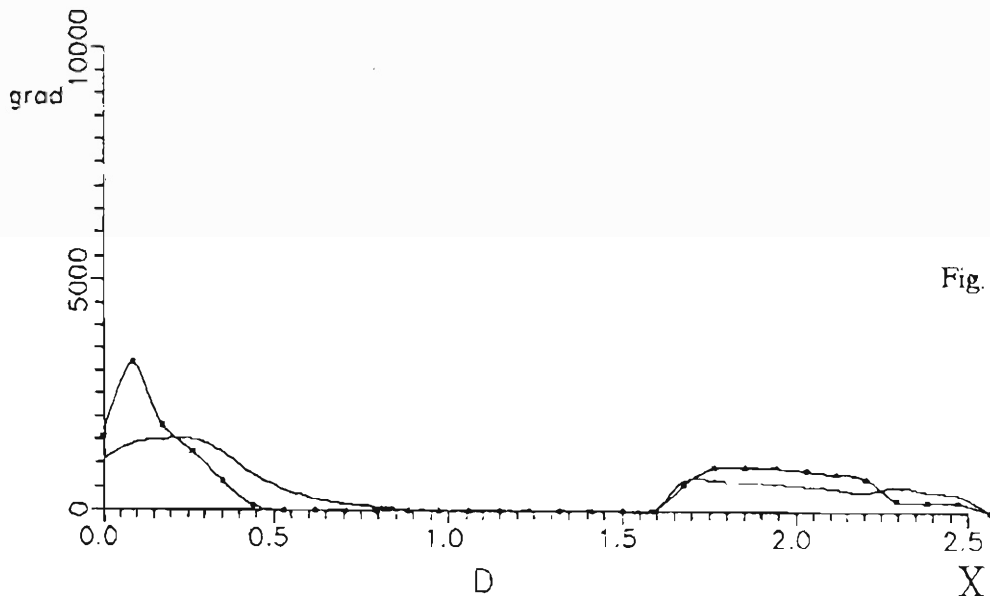


Fig. 2

2. **The formulation of the problem with combustion.** The following stage of the investigation into the given problem is that of considering combustion, or, more precisely, the process of the oil burnout from the fabric inside the furnace.

To describe such a model, one more equation must be added to system (1)–(5), so that the oil concentration might be calculated:

$$\frac{\partial c}{\partial t} + v \frac{\partial c}{\partial x} = -cAe^{-E/Ru} \quad (7)$$

with the corresponding initial and boundary conditions $c(x,0) = c(0,t) = c_0$

To the right-hand side of Eq.(6) a term regarding the burnout process should be added:

$$c(x,t)\Delta H A e^{-E/Ru}$$

Here: c is the remained oil concentration;

ΔH is the heat effect of the reaction;

A is the pre-exponent;

E is the activation energy;

R is the molar gas constant.

Since the real burnout process occurs rapidly and on a short fabric section, the solution of the corresponding problem has, in reality, a practically break. To facilitate calculation, in Eq. (7) an additional term

is introduced, with artificial viscosity: $\lambda_c \frac{\partial^2 c}{\partial x^2}$. Therefore a final form of the equation system for calculation of burnout process shows as

$$\rho c_p \frac{\partial u}{\partial t} = \lambda \frac{\partial^2 u}{\partial x^2} - v \rho c_p \frac{\partial u}{\partial x} - \frac{\epsilon_L \sigma (2 - \epsilon_L)}{\delta (1 - \epsilon_L)} u^4 + \frac{\epsilon_L \sigma}{\delta} T_k^4 + \frac{\epsilon_L \sigma}{\delta (1 - \epsilon_L)} q_0 - \frac{2\alpha}{\delta} (u - T_g) + c \Delta H A e^{-E/Ru},$$

$$\frac{\partial c}{\partial t} + v \frac{\partial c}{\partial x} + \lambda_c \frac{\partial^2 c}{\partial x^2} = -cAe^{-E/RT},$$

with corresponding boundary and initial conditions.

In the following figures the calculational variations for full burnout model are presented. In Fig. 3, there is given the temperature distribution along the fabric inside the furnace and the after-furnace chamber with regard to the burnout process. For the model taking into account the Stefan-Boltzman radiation (a continuous curve), a more abrupt temperature jump is characteristic, and for the model accounting for reflected heat fluxes (an asterisk curve) the jump is smoother, which better fits the reality.

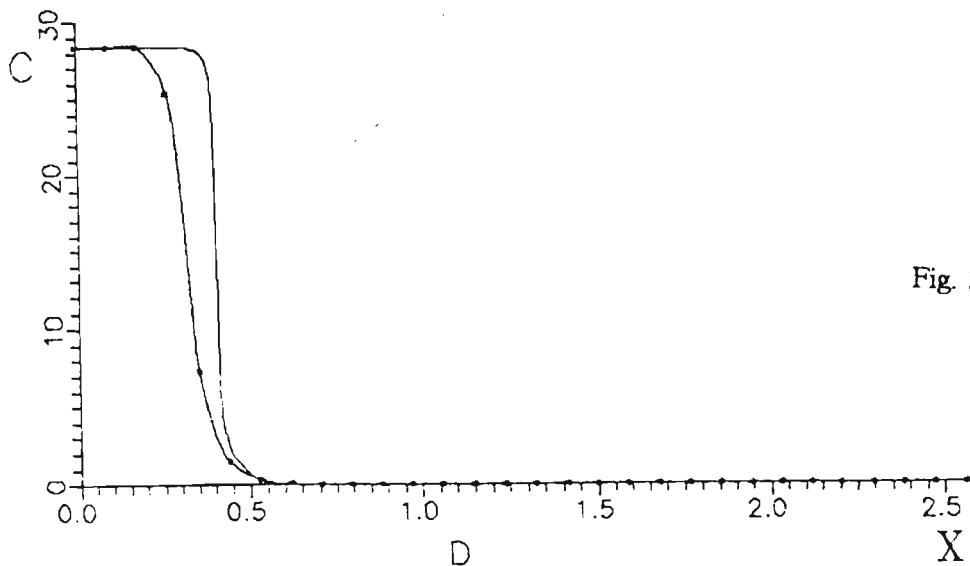
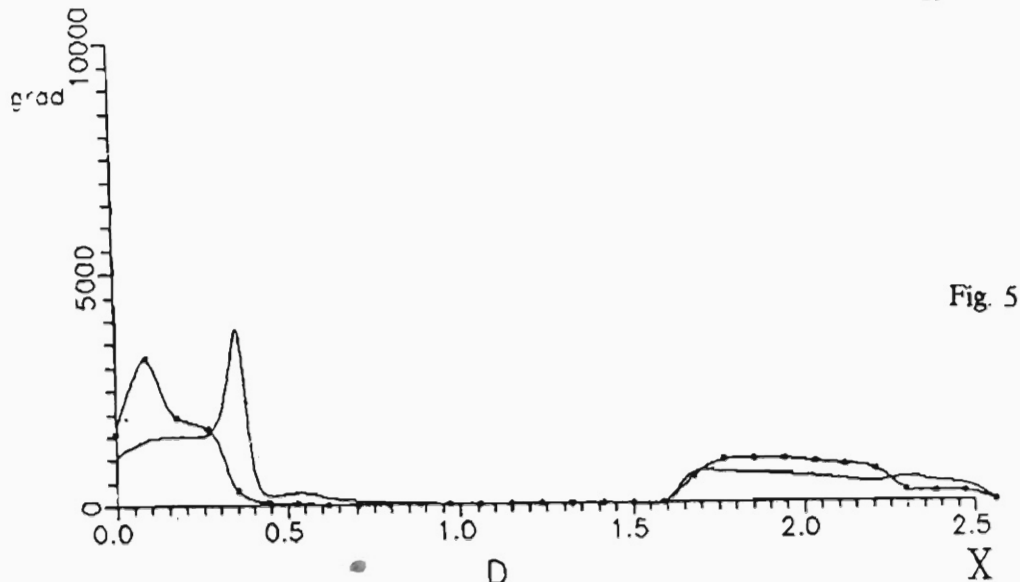
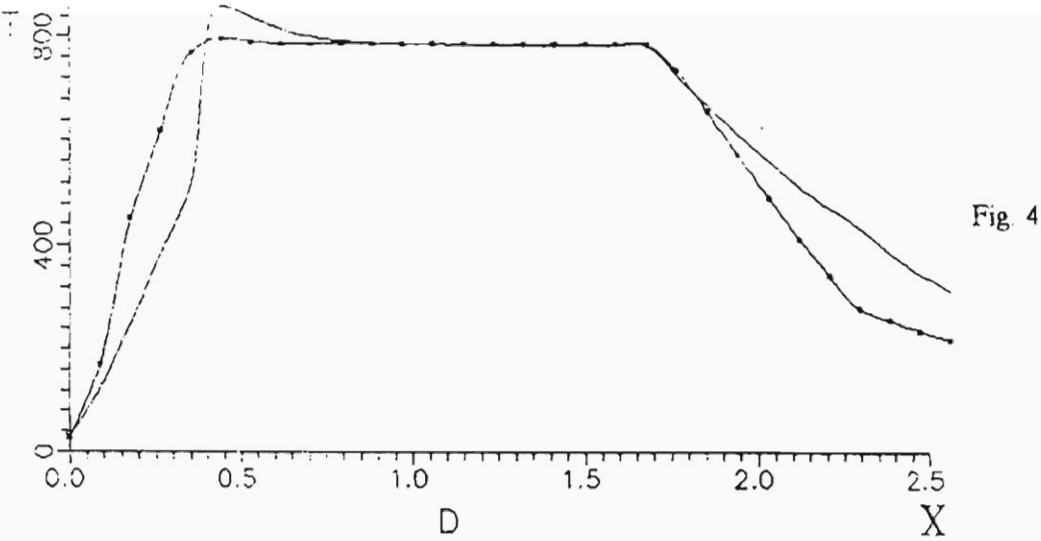


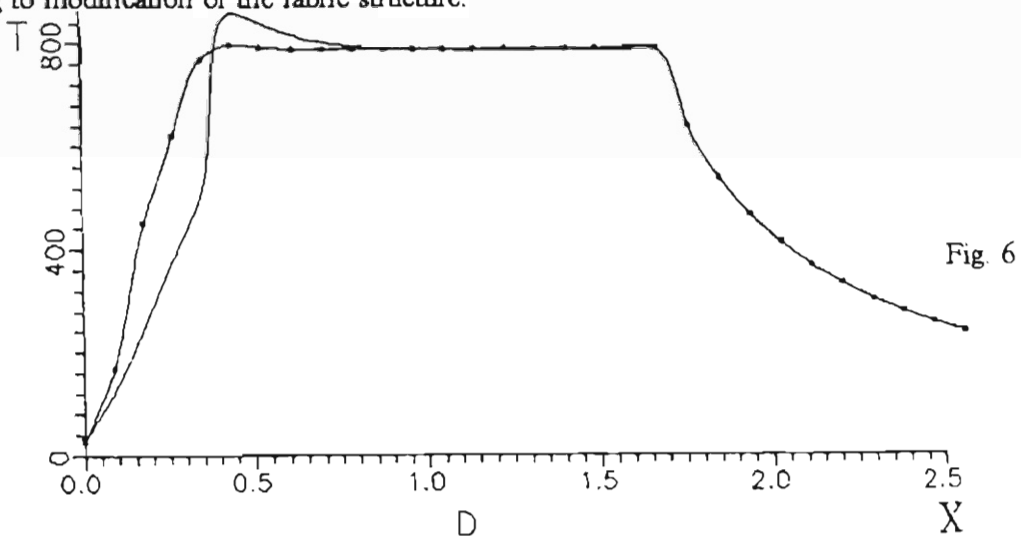
Fig. 3

In Fig. 4, there are depicted the curves of changes in oil concentration on the fabric for both the calculation models. The burnout process with reflexed heat fluxes proceeds several times slower, so the increase in combustion temperature is not large (near background).



In Fig. 5, there are presented the temperature gradients on the fabric, respectively, for the case when the abrupt jump of the temperature gradient occurring at the burnout moment is on the curve corresponding to the solution of problem with Stefan-Boltzman radiation conditions, and for the case where such a jump appears at the furnace inlet only, i.e. the case with reflected heat fluxes (an asterisk curve).

After the furnace outlet the temperature gradients reduce due to the presence of a special chamber, the temperature inside of which varies smoothly from 800°C to 100°C . In Figs. 6 and 7, there are shown the temperature and its gradient distributions in the absence of such an after-furnace chamber. To the steep temperature drop on the fabric at the furnace outlet there corresponds an abrupt gradient rise, which may cause such undesirable effect as appearance of internal stresses and deformations in the fabric material leading to modification of the fabric structure.



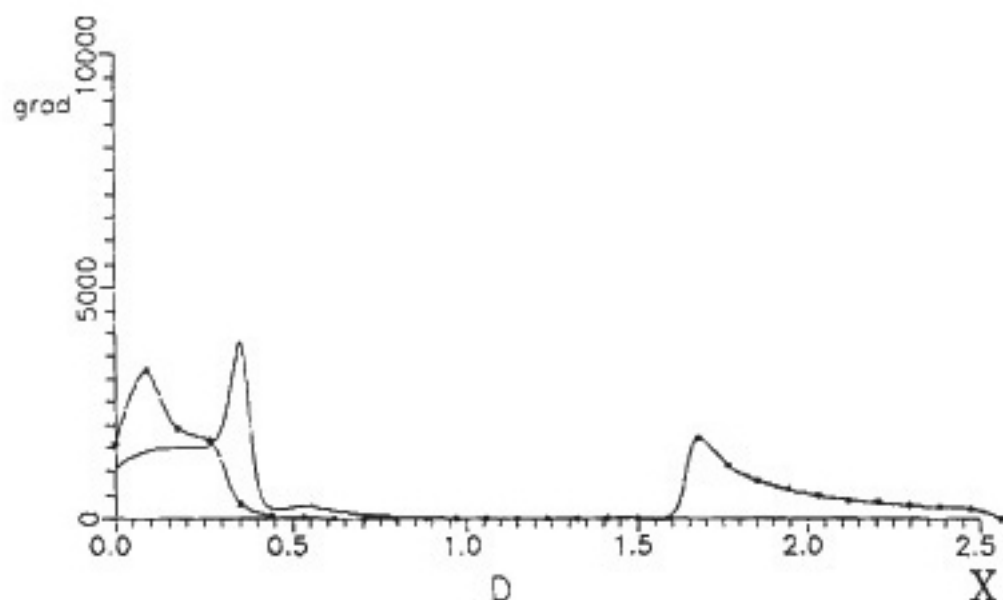


Fig. 7

3. The data for the calculations.

$\delta=0.002m$ is a thickness of fabric,

$D=1.16m$ is a length of furnace,

$T_0=850^\circ C$ is the temperature of the heaters at the bottom of the furnace,

$T_k=700^\circ C$ is the temperature at the top of the furnace.

$T_g=720^\circ C$ is the gas temperature in the furnace,

$\epsilon_L = 0.92$ is the emissivity of the fabric material,

$\epsilon_N = 0.8$ is the emissivity of the heater material,

$\rho=1100kg/m^3$, $c_p=690.J/kgK$, $\lambda=1.38W/mK$,

$c_0=28.387mol/m^3$ is the initial concentration,

$A=10^9/sec$, $\Delta H=1207 \cdot 10^7 j/mol$, $E=160 \cdot 10^3 kj/mol$, $\lambda_c=0.0001$.

References

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